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**THESIS**

AN ACCURACY ANALYSIS OF ARMY MATERIAL  
SYSTEM ANALYSIS ACTIVITY DISCRETE  
RELIABILITY GROWTH MODEL

by

Rio M. Thalieb

March 1988

Thesis Advisor: W. M. Woods

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Discrete Reliability Growth Model

by

Rio M. Thalieb  
Major, Indonesia Air Force

Submitted in partial fulfillment of the  
requirements for the degree of

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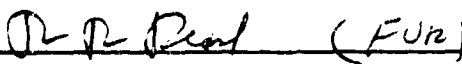
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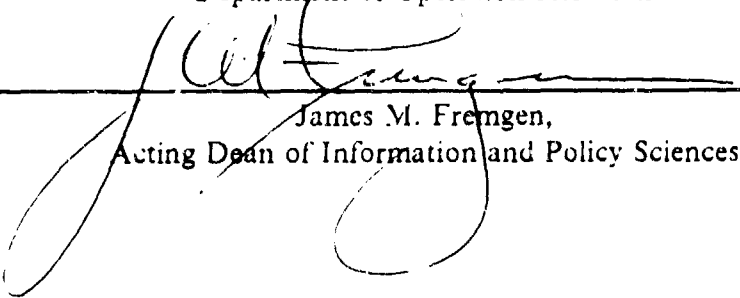
W. Max Woods, Thesis Advisor



Robert R. Read, Second Reader

 (FUR)

Peter Purdue, Chairman,  
Department of Operation Research



James M. Fremgen,  
Acting Dean of Information and Policy Sciences

## ABSTRACT

The accuracy of the discrete reliability growth model developed by Army Material System Analysis Activity (AMSAA) is analysed. The mean, standard deviation, and 95 percent confidence interval of the estimate of reliability resulting from simulating the AMSAA discrete reliability growth model are computed. The mean of the estimate of reliability from the AMSAA discrete reliability growth model is compared with the mean of the reliability estimate using the Exponential discrete reliability growth model developed at the Naval Postgraduate School and with the actual reliability which was used to generate test data for the replications in the simulations. The testing plan simulated in this study assumes that the mission tests (go-no-go) are performed until a predetermined number of failures occur at which time a modification is made. The main results are that the AMSAA discrete reliability growth model always performs well with concave growth patterns and has difficulty in tracking the actual reliability which has convex growth pattern or constant growth pattern when the number of failures specified equal to one.

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## I. INTRODUCTION

The test-analyze-and-fix scenario is frequently followed in order to achieve high reliability under current DOD design and development policies during early development. An item will usually be tested until it fails. The failure is analyzed to determine its cause, and what needs to be done to remove the cause of failure. Appropriate changes are made and more items are tested until the next failure occurs. After each modification to the item, it has a new reliability and after the  $K^{th}$  modification we are in the  $K^{th}$  reliability growth phase and all items tested in this phase have common reliability  $R_k$ . This procedure is repeated several times until the requirement for reliability is achieved. Through this procedure a reliability growth pattern is established. Reliability growth models have been developed to estimate reliability from phase to phase for this type of test program. One such model is the Army Material System Analysis Activity (AMSAA) Discrete Reliability Growth Model.

The purpose of this paper is to perform an accuracy analysis of the AMSAA discrete reliability growth model. Performance evaluation of the AMSAA discrete reliability growth model was done using monte carlo simulation to generate test data which in turn was used to exercise the AMSAA computer program to compute the estimate of the reliability for each phase. The reliability estimates obtained from the AMSAA model are compared with the actual reliability in a predetermined sequence of reliabilities which used to generate test data. In addition these values are compared with the reliability estimate obtained from the Exponential discrete reliability growth model which has been analyzed at the Naval Postgraduate School [Ref. 1, 2, and 3]. General description of the analysis used in this paper is described below :

For each phase,

- Assign value  $R_i$ , the reliability for  $i^{th}$  phase
- Specify  $F_i$ , the number of failures specify to stop the phase
- Generate  $N_i$ , the number of tests needed to obtained  $F_i$  failures
- Collect the test data,  $N_i$  and  $F_i$
- Compute  $\hat{R}_i$ , the estimate of  $R_i$
- Replicate this scenario 500 times
- Compute the sample mean  $\bar{\hat{R}}_i$  and sample standard deviation  $S_{\hat{R}_i}$

- Compute a 95% confidence interval for  $E[\hat{R}_i]$
- Compare  $\bar{R}_i$  with  $R_i$  in graphical form
- Compare  $\bar{R}_i$  with the estimate of reliability using the Exponential discrete reliability growth model with the same data
- Prepare appropriate graphs.

## II. AMSAA DISCRETE RELIABILITY GROWTH MODEL

The AMSAA discrete reliability growth model [Ref. 4] was developed by L. Crow in 1983 is based on a learning curve approach that had been mentioned by Duane [Ref. 5]. This model is applicable when items under test are scored as success or failure. The model is derived from a reliability growth model for continuous data, i.e., time to failure data. It is based on the Duane reliability growth concepts.

### A. INTERPRETATION OF LEARNING CURVE PROPERTY

Let  $C(t)$  be the cumulative failure rate,  $K(t)$  be the total number of failures by time  $t$ , where  $t$  is the cumulative test time. The ratio of  $K(t)$  and  $t$  is equal to  $C(t)$

$$\frac{K(t)}{t} = C(t)$$

Duane observed that  $\ln C(t)$  was linear when plotted against  $\ln t$ . That is,

$$\ln C(t) = \delta - \alpha \ln t$$

Duane expressed this relationship differently as follows :

$$\ln \frac{K(t)}{t} = \delta - \alpha \ln t$$

$$\ln \frac{K(t)}{t} + \alpha \ln t = \delta$$

$$\ln \frac{K(t)}{t^{1-\alpha}} = \delta$$

$$\frac{K(t)}{t^{1-\alpha}} = e^{\delta}$$

$$\text{let } 1 - \alpha = \beta$$

$$\text{and let } e^{\delta} = \lambda$$

$$K(t) = \lambda t^{\beta}$$

Consequently, if  $\ln C(t)$  is a linear function of  $\ln t$ , this implies  $K(t) = \lambda t^{\beta}$ , which is the learning curve property for  $K(t)$ . [Ref. 4 page 1].

## B. AMSAA DISCRETE RELIABILITY GROWTH MODEL DEVELOPMENT

The discrete reliability growth model developed at AMSAA uses attributes data. This model is described as follows :

$N_i$  = Number of trials for configuration  $i$ ,  $i = 1, 2, \dots, k$

$T_i$  = Cumulative number of trials through configuration  $i$

$$T_1 = N_1$$

$$T_2 = N_1 + N_2$$

In general :

$$T_i = N_1 + N_2 + N_3 + \dots + N_i$$

$M_i$  = Number of failures for configuration  $i$

$K_i$  = Cumulative number of failures through configuration  $i$

$$K_1 = M_1$$

$$K_2 = M_1 + M_2$$

In general :

$$K_i = M_1 + M_2 + M_3 + \dots + M_i$$

$E[K_i]$  = Expected value of  $K_i$  .

The model assumes that  $\log E[K_i]$  is linear when plotted against  $\log T_i$ . This implies  $E[K_i] = \lambda T_i^p$  . [Ref. 4 page 1 to 4].

$$E[K_1] = \lambda T_1^p = P_1 N_1$$

$$P_1 = \frac{\lambda T_1^p}{N_1}$$

$$E[K_2] = \lambda T_2^p = P_1 N_1 + P_2 N_2$$

$$\lambda T_2^p = \lambda T_1^p + P_2 N_2$$

$$P_2 N_2 = \lambda T_2^p - \lambda T_1^p$$

$$P_2 = \frac{\lambda T_2^p - \lambda T_1^p}{N_2}$$

In general :

$$P_i = \frac{\lambda T_i^p - \lambda T_{i-1}^p}{N_i} .$$

[Ref. 4 page 5 to 6].

### C. ESTIMATION PROCEDURE FOR AMSAA DISCRETE MODEL

The maximum likelihood estimates  $\hat{\lambda}$  and  $\hat{\beta}$  for  $\lambda$  and  $\beta$  were derived by AMSAA as follows :

Let,

$P_i$  = Probability of failure for configuration i

$\hat{P}_i$  = Estimated failure probability for configuration i

$R_i$  = Reliability for configuration i

$\hat{R}_i$  = Estimated reliability for configuration i

Then

$$\hat{P}_i = \frac{\hat{\lambda} T_i^{\hat{\beta}} - \hat{\lambda} T_{i-1}^{\hat{\beta}}}{N_i}$$

$$\hat{R}_i = 1 - \hat{P}_i$$

where maximum likelihood estimates of  $\hat{\lambda}$  and  $\hat{\beta}$  are values satisfying :

$$\sum_{i=1}^k \left\{ \frac{M_i}{[\lambda T_i^{\beta} - \lambda T_{i-1}^{\beta}]} - \frac{N_i - M_i}{[N_i - \lambda T_i^{\beta} + \lambda T_{i-1}^{\beta}]} \right\} [\lambda T_i^{\beta} \ln T_i - \lambda T_{i-1}^{\beta} \ln T_{i-1}] = 0$$

and,

$$\sum_{i=1}^k \left\{ \frac{M_i}{[\lambda T_i^{\beta} - \lambda T_{i-1}^{\beta}]} - \frac{N_i - M_i}{[N_i - \lambda T_i^{\beta} + \lambda T_{i-1}^{\beta}]} \right\} [T_i^{\beta} - T_{i-1}^{\beta}] = 0.$$

[Ref. 4 page 10 to 12].

In this paper  $\hat{\lambda}$  and  $\hat{\beta}$  were computed using the AMSAA computer program.

### III. EXPONENTIAL DISCRETE RELIABILITY GROWTH MODEL

The Exponential discrete reliability growth model has been analyzed at the Naval Postgraduate School in two theses [Ref. 1, 2], and by Corcoran and Read [Ref. 3], where Corcoran and Read have compared several popular reliability growth models. This model serves as a model comparison to the AMSAA discrete reliability growth model. The Exponential discrete reliability growth model uses only attribute data. It does not require any assumption about the distribution of the time to failure. This model is described briefly as follows :

Let :

$R_i$  = The reliability of the component in phase  $i$

$R_i = 1 - \exp\{-(\alpha + \beta i)\}$  where  $i = 0, 1, 2, \dots$

$i = 0$  means the phase prior to any modification

The parameter estimates  $\hat{\alpha}_i$  and  $\hat{\beta}_i$  of  $\alpha$  and  $\beta$  for phase  $i$  are computed using linear regression methods and an unbiased estimator for  $(\alpha + \beta i)$

$F_i$  = the total number of failure during phase  $i$

$N_{j,i}$  = the number of tests between the  $(j-1)^{th}$  failure and  $j^{th}$  failure, including the  $j^{th}$ , in phase  $i$

$j = 1, 2, 3, \dots, F_i$

$Y_{ji}$  = unbiased estimator of  $(\alpha + \beta i)$  using  $j^{th}$  sequence test in phase  $i$

An unbiased estimator  $Y_{ji}$  for  $(\alpha + \beta i)$  [Chernoff and Woods 1965] is known to be :

$$Y_{ji} \equiv (\alpha + \beta i) = \begin{cases} 0 & \text{if } N_{j,i} = 1 \\ 1 + \frac{1}{2} + \dots + \frac{1}{N_{j,i} - 1} & \text{if } N_{j,i} \geq 2 \end{cases}$$

for  $i = 0, 1, 2, \dots$  and  $j = 1, 2, 3, \dots, F_i$ .

Since  $N_{1,i}, N_{2,i}, \dots, N_{F_i,i}$  are independent random variables, then :



$\bar{Y}_i = \frac{(Y_{1,i} + Y_{2,i} + \dots + Y_{F,i})}{F_i}$  is also an unbiased estimates for  $(\alpha + \beta i)$ .

The least square estimates  $\hat{\alpha}_i$  and  $\hat{\beta}_i$  for  $\alpha$  and  $\beta$  at phase  $i$  are :

$$\hat{\beta}_i = \frac{\sum_{j=0}^i (j-i) \bar{Y}_j}{\sum_{j=0}^i (j-i)^2} \quad \text{for } i = 1, 2, 3, \dots$$

and,

$$\hat{\alpha}_i = \bar{Y} - \hat{\beta}_i \bar{i} \quad \text{for } i = 1, 2, 3, \dots$$

where :

$$\bar{Y} = \frac{(Y_0 + Y_1 + \dots + Y_i)}{(i+1)}$$

$$\bar{i} = \frac{(0 + 1 + 2 + \dots + i)}{(i+1)}.$$

By using  $\hat{\alpha}_i$  and  $\hat{\beta}_i$  the estimate of reliability for every phase  $i$  can be computed as follows :

$$\hat{R}_i = 1 - \text{EXP}\{-(\hat{\alpha}_i + \hat{\beta}_i i)\} \quad \text{for } i = 1, 2, 3, \dots$$

The estimate of reliability for the original version of the component  $\hat{R}_0$  is given by :

$$\hat{R}_0 = 1 - \text{EXP}\{-\bar{Y}_0\}.$$

[Ref. 6 page 3-1 to 3-3].

In this paper the value of the mean regression estimate  $\bar{\hat{R}}_i$  of reliability and the value of standard deviation of the estimate of reliability  $S_{\hat{R}_i}$  were obtained from a computer program used in J. Chandler thesis [Ref. 2]. The equations for computing the reliability growth values  $\hat{R}_i$  are easily solved using a hand-held calculator.

#### IV. MONTE CARLO SIMULATIONS

Since the AMSAA model is interested in the number of trials until the  $r^{\text{th}}$  failure occurs, the Monte Carlo simulation generates random variable using an algorithms developed by Fishman 1978 [Ref. 7], and a subroutine from The New Naval Postgraduate School Random Number Package LLRANDOMII 1981 [Ref. 8] as a random number generator for real uniform from 0 to 1. Given  $p$ , the probability of failure, and  $r$ , the number of failure for every phase, the computer simulation generated the number of trials until the  $r^{\text{th}}$  failure. Specifically let  $X$  be the random variable of interest, the number of trials until the  $r^{\text{th}}$  failure, then  $X$  is called negative binomial random variable with parameter  $r$  and  $p$ . The probability function for  $X$  is,

$$P_r(k) = \binom{k-1}{r-1} p^r q^{k-r} \quad k = r, r+1, r+2, \dots \quad r > 0$$

The Algorithms for Computer Simulation :

1. let  $A$  and  $B$  be double precision variables
2.  $w = (p)^r$
3. If  $r \leq 0$ ,  $(1-p) \leq 0$ ,  $(1-p) \geq 1$ ,  $w \leq 0$ ,  $w \geq 1$  go to 9
4.  $X = r$ ,  $A = w$ ,  $B = w$  and  $\theta = (1-p)(r-1)$
5. Generate  $U$ , uniform random number from 0 to 1
6. If  $U \leq A$  or  $A > 0.999999$  or  $B < 0.000001$ , go to 10
7.  $X = X + 1$ ,  $B = B(6/X + (1-p))$  and  $A = A + B$
8. Go to 4
9. Print error message and stop
10. Continue.

[Ref. 7 page 354].

## V. TEST PROCEDURE

The AMSAA model is evaluated using eight different sets of reliability values (the actual growth pattern) and two different sets of inputs of number of failure per phase. This gives a total of sixteen cases. Table 1 describes all 16 cases. The set of reliability values for case  $i$  is the same as that for case  $i + 8$ ,  $i = 1, 2, \dots, 8$ . For cases 1 through 8 the number of failures per phase are equal to one and for cases 9 through 16 the number of failures per phase are equal to three. The diagram in Figure 1 summarizes the simulation procedure and the consequent analysis.

Table 1. ACTUAL RELIABILITY FOR 16 CASES

PHASE	CASE NUMBERS							
	1,9	2,10	3,11	4,12	5,13	6,14	7,15	8,16
1	.600463	.408036	.899215	.408036	.408036	.408036	.404786	.400000
2	.600463	.408036	.899215	.804273	.804273	.691333	.598442	.430000
3	.600463	.408036	.899215	.950990	.894416	.804273	.796763	.480000
4	.600463	.408036	.899215	.975249	.899963	.603542	.796763	.540000
5	.600463	.408036	.899215	.990040	.899963	.600463	.802460	.610000
6	.600463	.408036	.899215	.990040	.899963	.755720	.802460	.700000
7	.600463	.408036	.899215	.990040	.899963	.849243	.857802	.800000
8	.600463	.408036	.899215	.990040	.899963	.894416	.902960	.900000
9	.600463	.408036	.899215	.990040	.899963	.903636	.902960	.950000
10	.600463	.408036	.899215	.990040	.899963	.903636	.902960	.990000

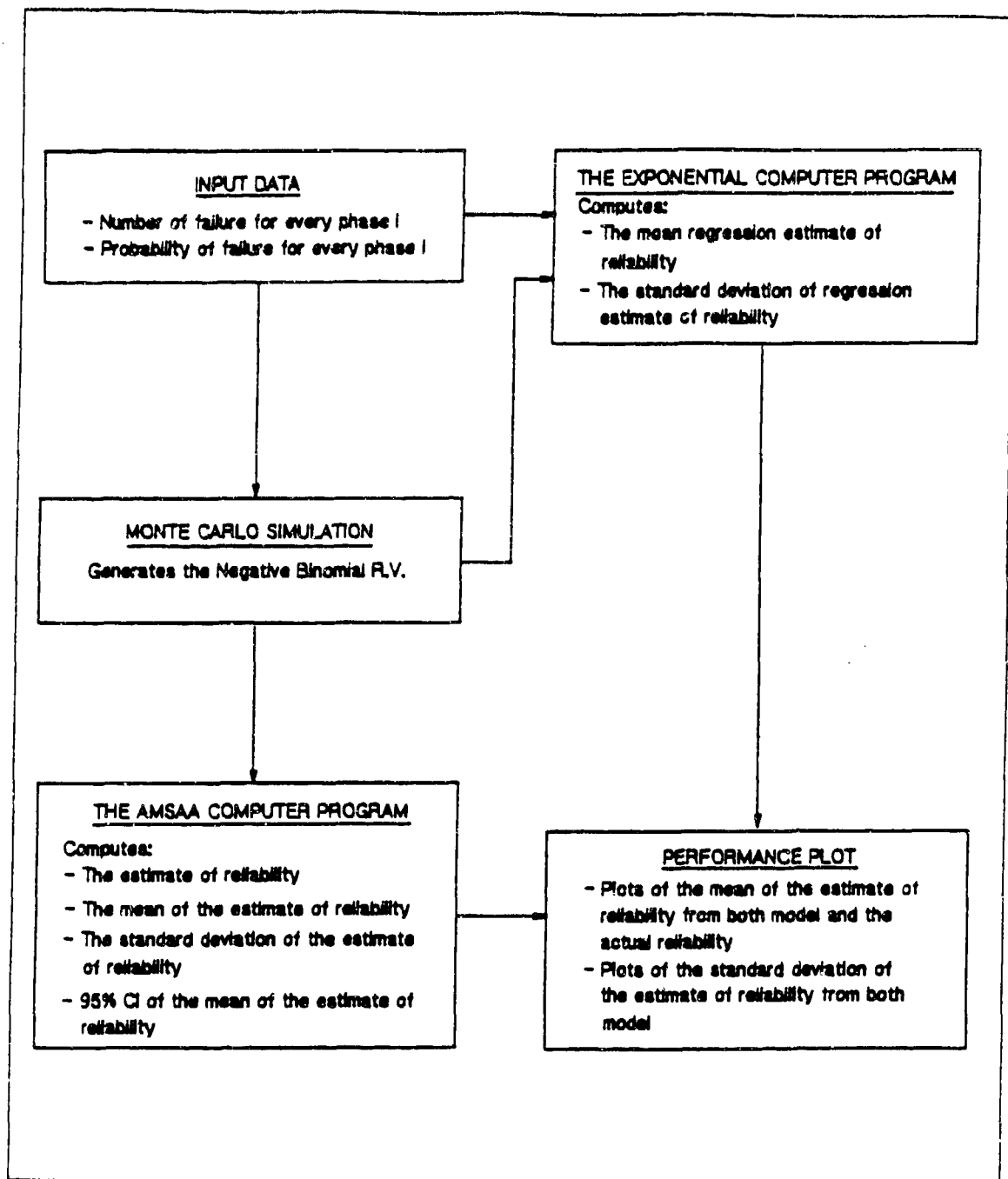


Figure 1. Block diagram of the analysis

## VI. ANALYSIS PROCEDURE

### A. ACCURACY

Figures 4 through 35 in the Appendix provide a visual display of the AMSAA discrete reliability growth model accuracy by comparing the growth line for the AMSAA with the actual reliability growth pattern. These graphs also provide plots for the Exponential discrete reliability growth model using the same input data as that used in the AMSAA discrete reliability growth model.

### B. VARIABILITY

In addition to the tracking ability of the reliability point estimates  $\bar{\hat{R}}_i$ , the user is also interested in the variability of  $\hat{R}_i$ . Five hundred replications were run for each of the 16 cases and each of the 10 phases, this provided :

$$\bar{\hat{R}}_i = \frac{1}{500} \sum_{i=1}^{500} \hat{R}_i$$

and,

$$S_{\hat{R}_i} = \sqrt{\frac{\sum_{i=1}^{500} (\hat{R}_i - \bar{\hat{R}}_i)^2}{499}}$$

for  $i = 1, 2, \dots, 10$  for each of the 16 cases.

The algorithm used to compute the mean and standard deviation is developed by Miller 1982 [Ref. 9 page 17 to 19]. Standard deviation of the reliability estimates from both the AMSAA and the Exponential model are plotted.

### C. CONFIDENCE INTERVAL

A 95% two sided confidence interval for  $E[\hat{R}_i]$  is computed for each model for all 16 cases. The equation used for these confidence limits are as follows :

$$U_i = \bar{R}_i + \frac{(1.96)S_{\hat{R}_i}}{\sqrt{500}}$$

and,

$$L_i = \bar{R}_i - \frac{(1.96)S_{\hat{R}_i}}{\sqrt{500}}$$

for  $i = 1, 2, \dots, 10$  for each of the 16 cases.

## VII. RESULTS AND CONCLUSIONS

The results for all 16 cases can be seen in the Appendix. The case number appears at the table caption or at the figure caption. All of the results for each case are divided into two categories, i.e., the tabulated statistic and performance plot. In this chapter Table 2, Figure 2, and Figure 3 are explained as an example of the result from case 1 of data set.

### A. TABULATED STATISTICS

Table 2 indicates that testing was done until one failure occurred after which a change in the item was made. The actual reliability growth values for each of the 10 phases was constant at 0.60043, i.e., no growth actually occurred. It is important to simulate this case in order to examine the ability of the growth model to detect no growth. Some reliability growth models have a built in assumption that some growth always takes place after a design change.

The values of  $\bar{R}_i$  for  $i = 1, 2, \dots, 10$  are given in column 4 for the AMSAA model and in column 8 for the Exponential model, thus for phase 7, the AMSAA model yielded  $\bar{R}_7 = 0.580187$  and the Exponential model yielded a value of 0.525144. The corresponding values of the standard deviation are 0.124669 and 0.261854 for the AMSAA and the Exponential model respectively.

### B. PERFORMANCE PLOT

Figure 1 is a plot of  $\bar{R}_i$  versus  $i$  for the AMSAA and Exponential models. It also displays a plot of the actual reliabilities  $R_i$ . Figure 2 is a plot of standard deviation for case 1 for both the AMSAA and the Exponential model.

Table 2. STATISTICS FOR CASE 1

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL.	ACTUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	1	.600463	.319489	.311700	.346811	.292167	.436848	.387013
2	1	.600463	.485770	.147713	.498718	.472823	.459899	.382840
3	1	.600463	.530347	.112104	.540173	.520521	.482593	.316476
4	1	.600463	.553948	.103685	.563037	.544860	.515253	.292478
5	1	.600463	.569518	.106661	.578867	.560169	.509663	.282941
6	1	.600463	.580187	.114242	.590201	.570173	.514700	.271440
7	1	.600463	.587260	.124669	.598187	.576332	.525144	.261854
8	1	.600463	.592922	.134212	.604686	.581158	.529729	.245836
9	1	.600463	.597331	.143335	.609895	.584767	.542551	.240164
10	1	.600463	.600715	.151730	.614014	.587415	.550677	.219819



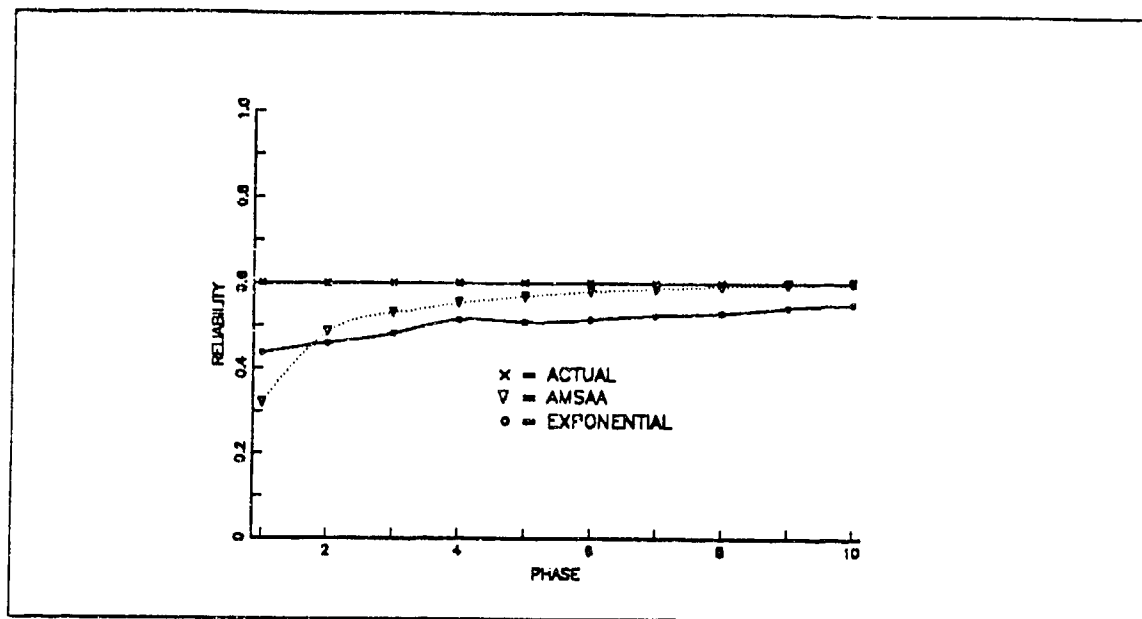


Figure 2. The reliability growth pattern comparison plot case 1

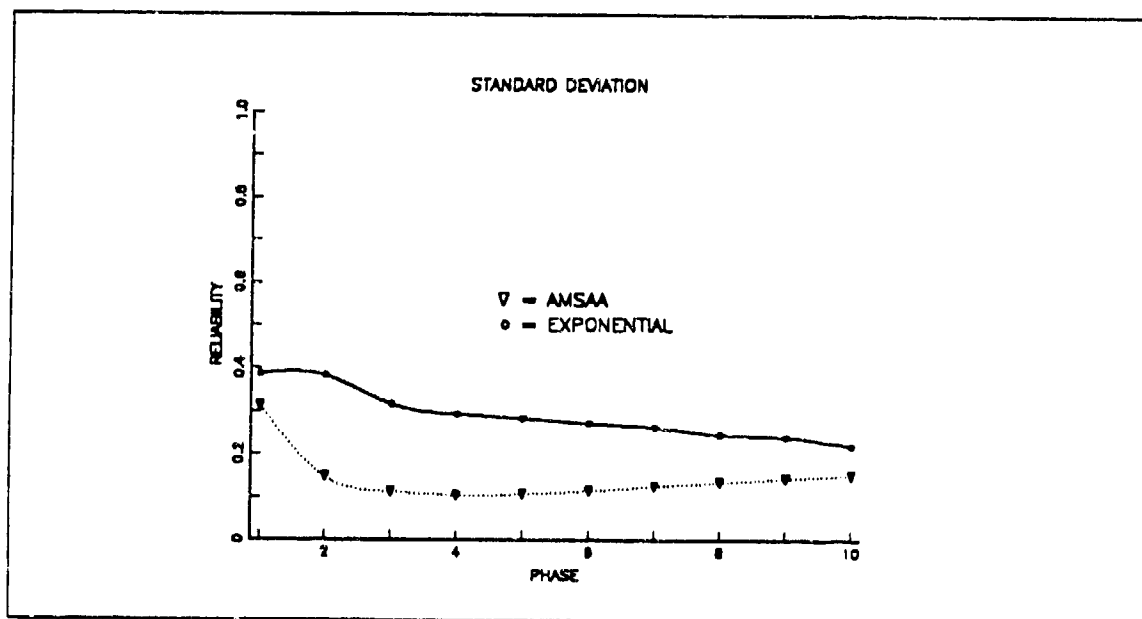


Figure 3. The standard deviation comparison plot case 1

### **C. SUMMARY AND CONCLUSIONS**

To analyze the test results for cases 1 through 16, all were divided into categories, i.e., constant growth pattern, concave with rapid growth pattern, concave and convex growth pattern, convex growth pattern (see Appendix).

#### **1. Constant Growth Pattern**

The AMSAA model didn't track the actual reliability too well for cases 1, 2, and 3 (the number of failure per phase was set equal to one). The AMSAA developed a concave growth pattern, even though in these cases the actual reliability was constant. Furthermore for case 3 the AMSAA model performance became worse since it had decreasing pattern and went below the actual reliability at phase 10. However when the number of failure increased to three, the AMSAA model tracked the actual reliability quite well. The mean of the estimate of reliability was close to the actual reliability, and the standard deviation of the estimate of reliability was very small.

#### **2. Concave with Rapid Growth Pattern**

This type of actual reliability growth pattern is represented in cases 4, 5, 12, and 13. The AMSAA model performed well in tracking actual reliability growth, especially for case 4, case 5, and case 13, where it is close to the actual reliability with very small standard deviation of the estimate of reliability. For case 12, the AMSAA model for some reason could not track the actual reliability very well. It performed almost constant growth, with a small decrease out through phase 10. This is a strange phenomena. This case was run several times with the same result.

#### **3. Concave and Convex Growth Pattern**

The AMSAA model has a problem tracking reliability growth pattern established in cases 6, 7, 14, and 15. The AMSAA model seems to display a concave growth pattern, it could not track the actual reliability which has a concave followed by a convex growth pattern. This is probably because the cumulative assumption inherent in the AMSAA model does not work well when the reliability growth has a convex growth pattern.

#### **4. Convex Growth Pattern**

The AMSAA model also had difficulty in tracking the actual reliability growth pattern for cases 8 and 16, it just performed concave growth pattern for both cases. The standard deviation of the estimate of reliability was good in these cases.

#### **5. Summary**

It appears that the AMSAA discrete reliability growth model is more appropriate for reliability growth pattern that has the following characteristics :

1. Concave with rapid growth pattern
2. Constant growth pattern with number of failure specified more than one.

It appears that the user should be wary of using the AMSAA discrete reliability growth model when the actual reliability growth has the following characteristics :

1. Constant growth pattern with number of failures specified equal to one
2. Concave growth followed by convex growth pattern
3. Convex growth pattern.

Also the user should note that other discrete reliability growth models are available which for some growth pattern performed better than the AMSAA model and which can be programmed on a hand-held calculator.

# APPENDIX SIMULATION RESULTS : CASE 1 TO CASE 16

Table 3. STATISTICS FOR CASE 1

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL.	ACTUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	1	.600463	.319489	.311700	.346811	.292167	.436848	.387013
2	1	.600463	.485770	.147713	.498718	.472823	.459899	.382840
3	1	.600463	.530347	.112104	.540173	.520521	.482593	.316476
4	1	.600463	.553948	.103685	.563037	.544860	.515253	.292478
5	1	.600463	.569518	.106661	.578867	.560169	.509663	.282941
6	1	.600463	.580187	.114242	.590201	.570173	.514700	.271440
7	1	.600463	.587260	.124669	.598187	.576332	.525144	.261854
8	1	.600463	.592922	.134212	.604686	.581158	.529729	.245836
9	1	.600463	.597331	.143335	.609895	.584767	.542551	.240164
10	1	.600463	.600715	.151730	.614014	.587415	.550677	.219819

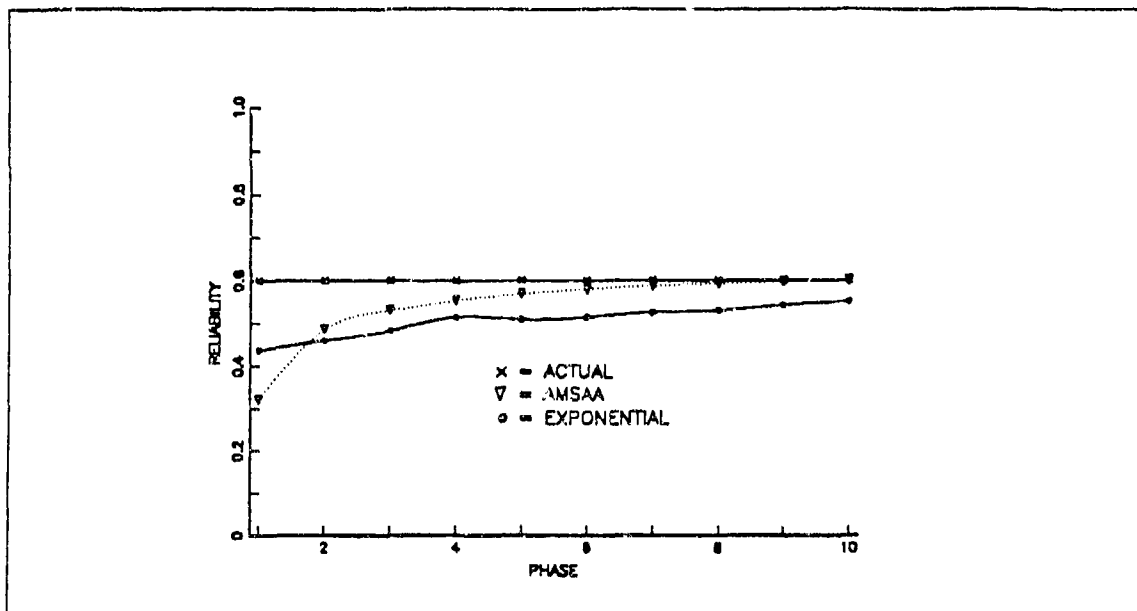


Figure 4. The reliability growth pattern comparison plot case 1

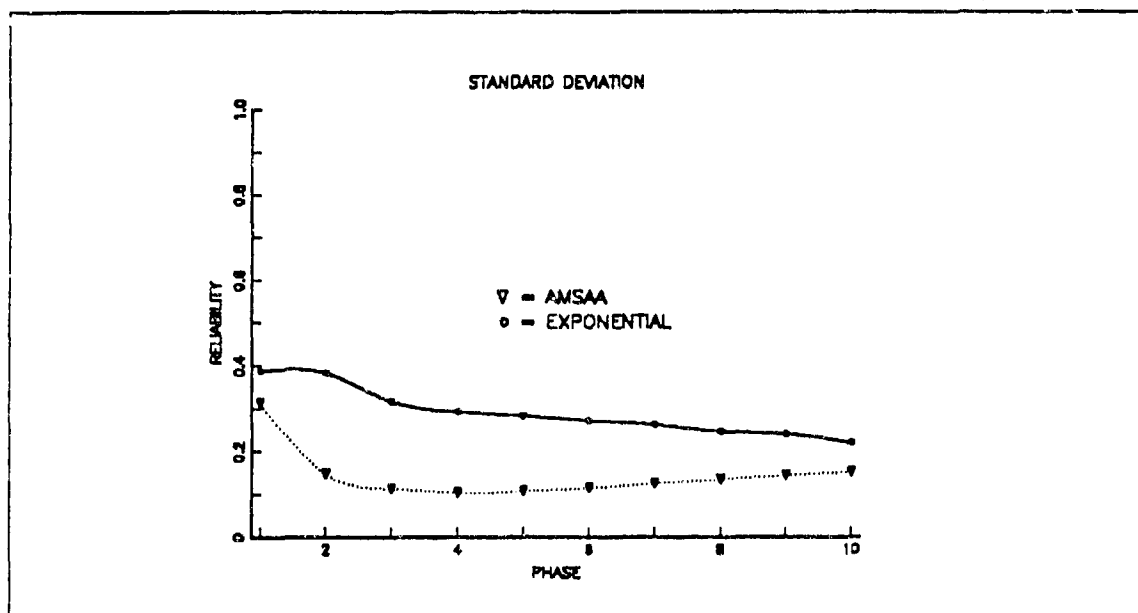


Figure 5. The standard deviation comparison plot case 1

Table 4. STATISTICS FOR CASE 2

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of TIAL	AC- TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	1	.403086	.416346	.365885	.448417	.384274	.277941	.350039
2	1	.403086	.462381	.232197	.482733	.442028	.313160	.354467
3	1	.403086	.459150	.173415	.474351	.443950	.345541	.294778
4	1	.403086	.448116	.133658	.459831	.436400	.350302	.278155
5	1	.403086	.431723	.109715	.441340	.422106	.332242	.274627
6	1	.403086	.413006	.110047	.422652	.403360	.342188	.266326
7	1	.403086	.394704	.128997	.406011	.383397	.340337	.252991
8	1	.403086	.376635	.157304	.390424	.362847	.348798	.246473
9	1	.403086	.360838	.186914	.377221	.344454	.359013	.239996
10	1	.403086	.346454	.215615	.365353	.327554	.380852	.232067

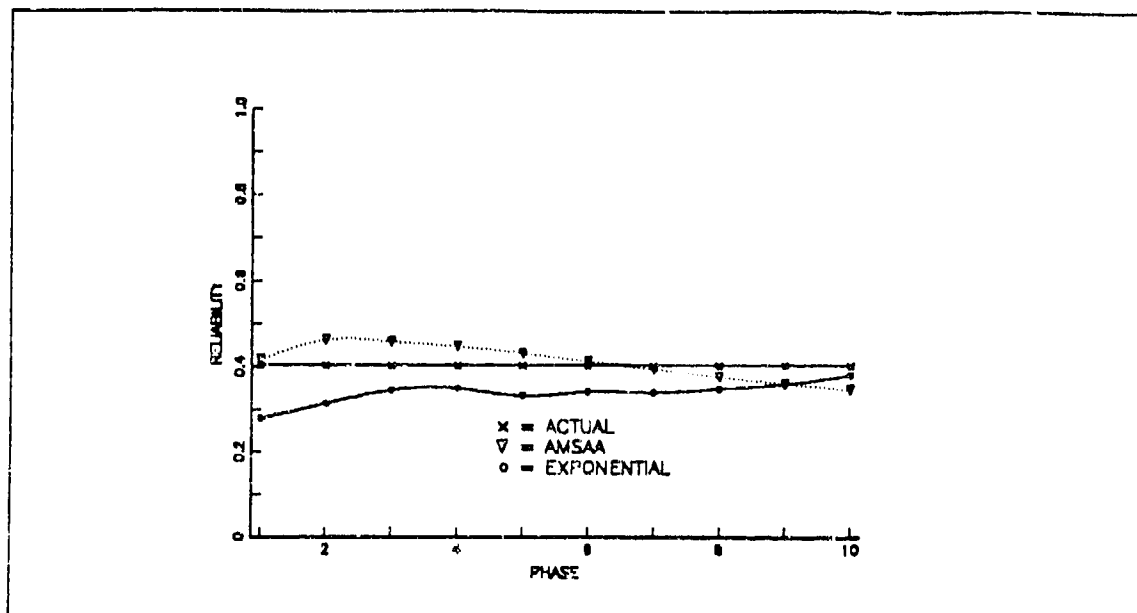


Figure 6. The reliability growth pattern comparison plot case 2

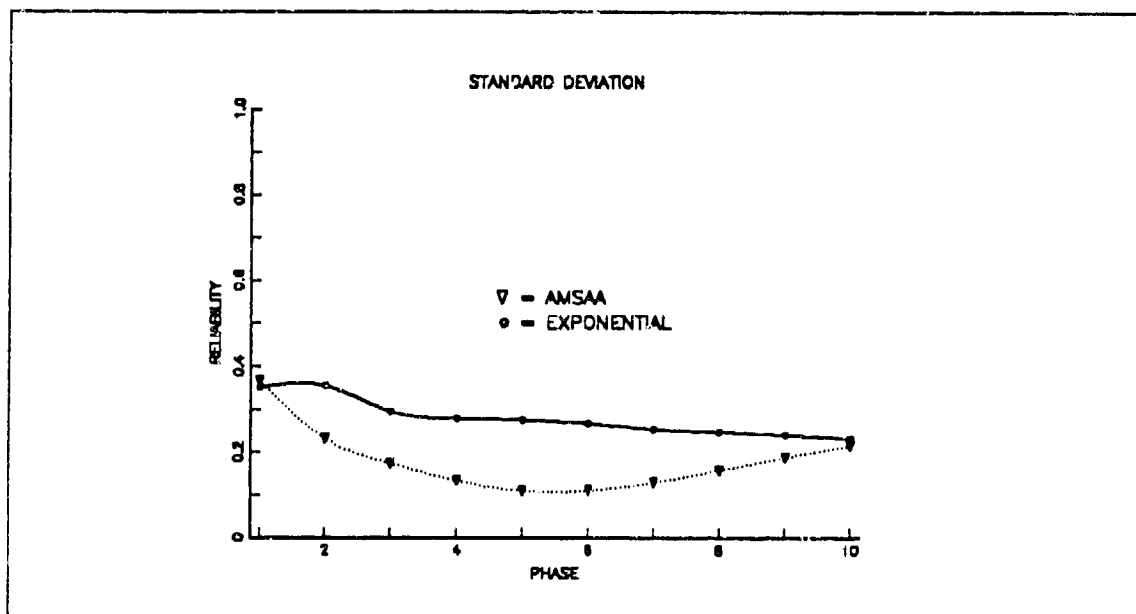


Figure 7. The standard deviation comparison plot case 2

Table 5. STATISTICS FOR CASE 3

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL.	AC-TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	1	.899215	.397305	.339065	.427025	.367585	.784433	.293429
2	1	.899215	.764665	.098591	.773307	.756023	.797078	.288337
3	1	.899215	.839154	.060455	.844453	.833855	.822922	.225154
4	1	.899215	.868815	.046403	.872882	.864747	.837676	.186795
5	1	.899215	.885368	.041623	.889016	.881719	.834702	.192540
6	1	.899215	.896082	.039685	.899561	.892603	.852273	.159859
7	1	.899215	.903360	.039227	.906798	.899921	.858921	.134809
8	1	.899215	.908869	.039064	.913293	.905444	.869732	.120620
9	1	.899215	.913205	.038957	.916620	.909790	.870845	.119773
10	1	.899215	.916762	.038813	.920164	.913360	.876847	.109467



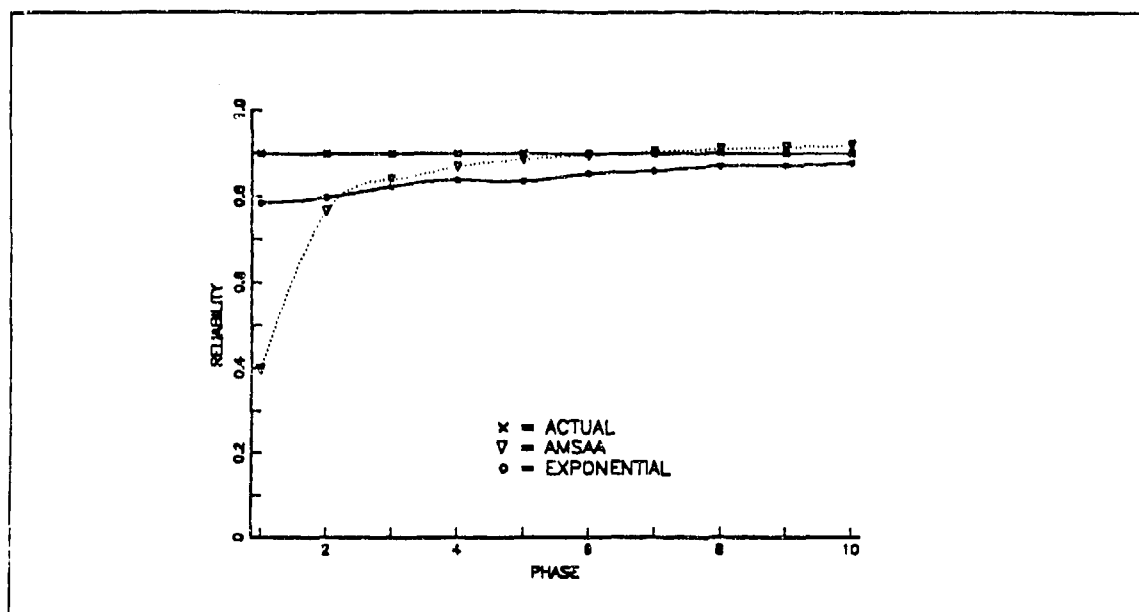


Figure 8. The reliability growth pattern comparison plot case 3

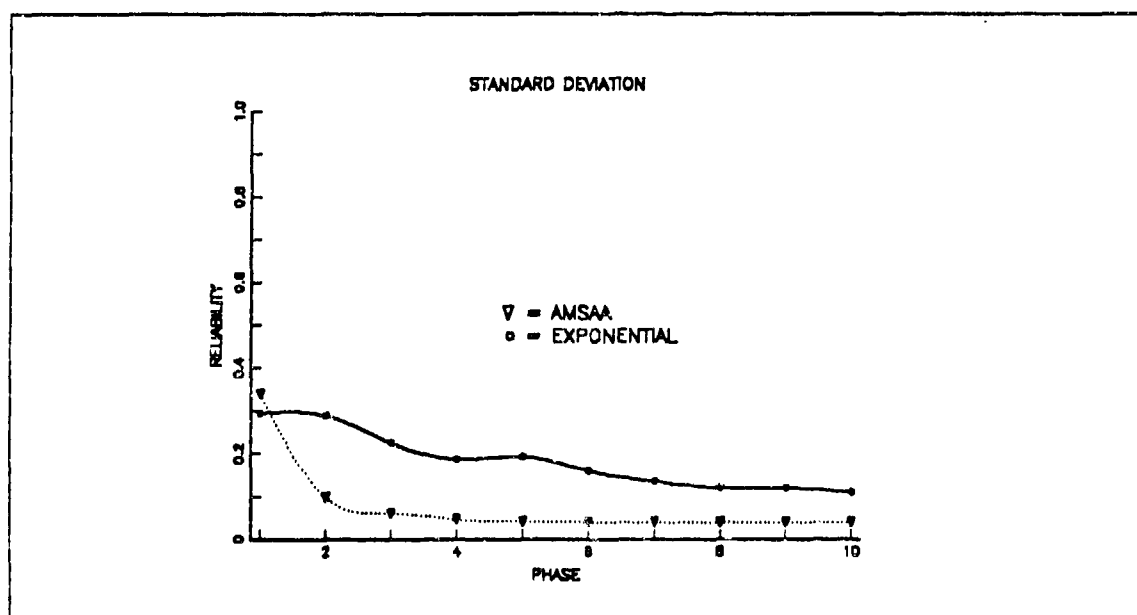


Figure 9. The standard deviation comparison plot case 3

Table 6. STATISTICS FOR CASE 4

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL.	ACTUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	1	.403086	.247188	.301170	.273586	.220789	.277941	.350039
2	1	.804723	.831764	.062844	.837272	.826255	.692599	.334232
3	1	.950990	.923927	.033311	.926847	.921007	.900547	.166031
4	1	.975249	.960687	.019283	.962377	.958997	.960575	.075075
5	1	.990040	.978110	.013216	.979269	.976952	.985843	.022317
6	1	.990040	.985506	.011104	.986480	.984533	.993530	.022859
7	1	.990040	.988832	.009410	.989657	.988007	.993346	.011663
8	1	.990040	.990717	.009306	.991532	.991066	.994365	.010904
9	1	.990040	.991889	.009384	.992711	.991066	.994678	.009990
10	1	.990040	.992741	.008820	.993514	.991968	.994922	.009867

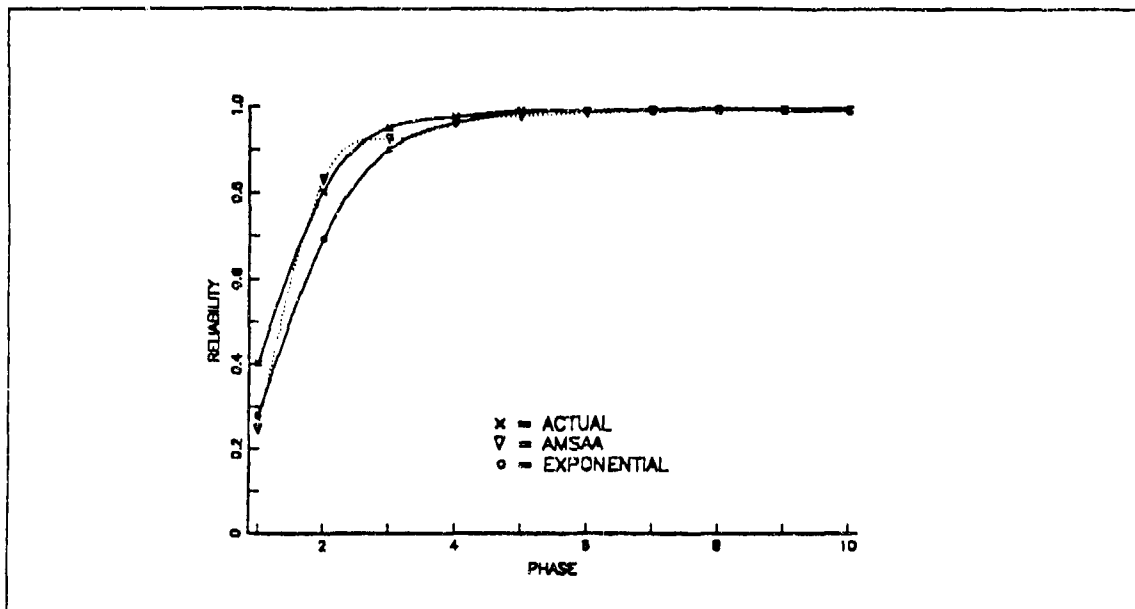


Figure 10. The reliability growth pattern comparison plot case 4

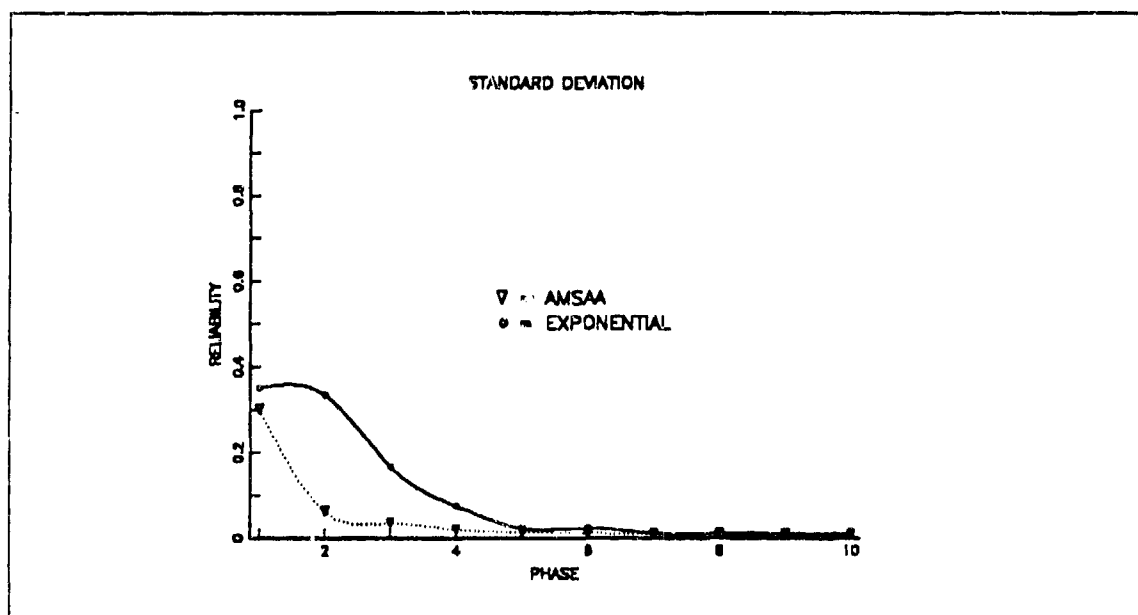


Figure 11. The standard deviation comparison plot case 4

Table 7. STATISTICS FOR CASE 5

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL	ACTUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	1	.403086	.304970	.319788	.333001	.276939	.277941	.350039
2	1	.804723	.716657	.099573	.725385	.707929	.692599	.334232
3	1	.894416	.811904	.063882	.817504	.806305	.816604	.229131
4	1	.899963	.854719	.050648	.859159	.850280	.857286	.180176
5	1	.899963	.877544	.045115	.881498	.873589	.854340	.201203
6	1	.899963	.891553	.042322	.895262	.887843	.852018	.233083
7	1	.899963	.900743	.041174	.904352	.897134	.883001	.196564
8	1	.899963	.907577	.040321	.911111	.904042	.880019	.208935
9	1	.899963	.912850	.039735	.916332	.909367	.886436	.092224
10	1	.899963	.917156	.038932	.920568	.913743	.889895	.215450

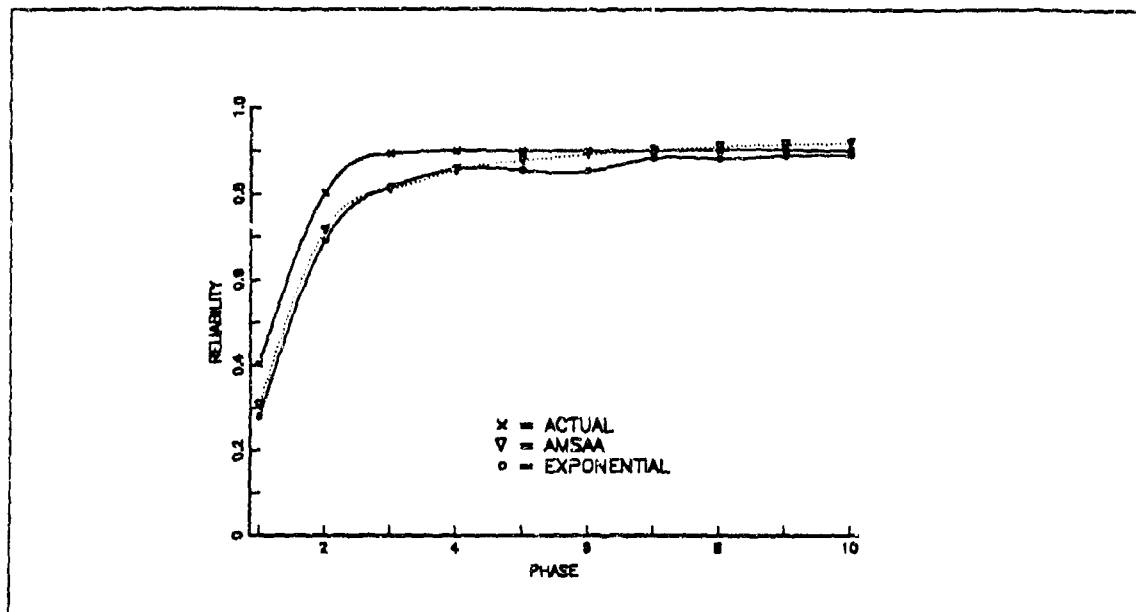


Figure 12. The reliability growth pattern comparison plot case 5

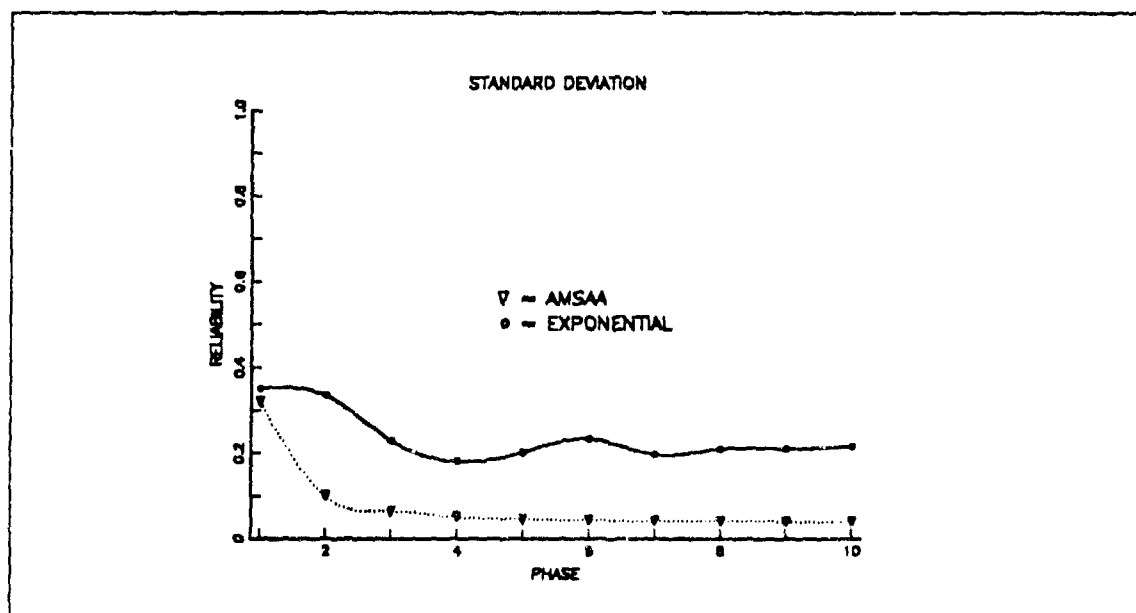


Figure 13. The standard deviation comparison plot case 5

Table 8. STATISTICS FOR CASE 6

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL	AC-TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	1	.403086	.224899	.271434	.248692	.201107	.277941	.350039
2	1	.691333	.630239	.101236	.639112	.621365	.566627	.371685
3	1	.804723	.728316	.078056	.735158	.721474	.702538	.282617
4	1	.603542	.768103	.070584	.774289	.761916	.649254	.242711
5	1	.600463	.787819	.064749	.793495	.782144	.609683	.240854
6	1	.755710	.805681	.061914	.811108	.800254	.674674	.217185
7	1	.849243	.823900	.062269	.829358	.818442	.763961	.167637
8	1	.894416	.841536	.063679	.847117	.835954	.830485	.122824
9	1	.903636	.855559	.064934	.861250	.849867	.865976	.105627
10	1	.903636	.866080	.064945	.871772	.860387	.889152	.084728

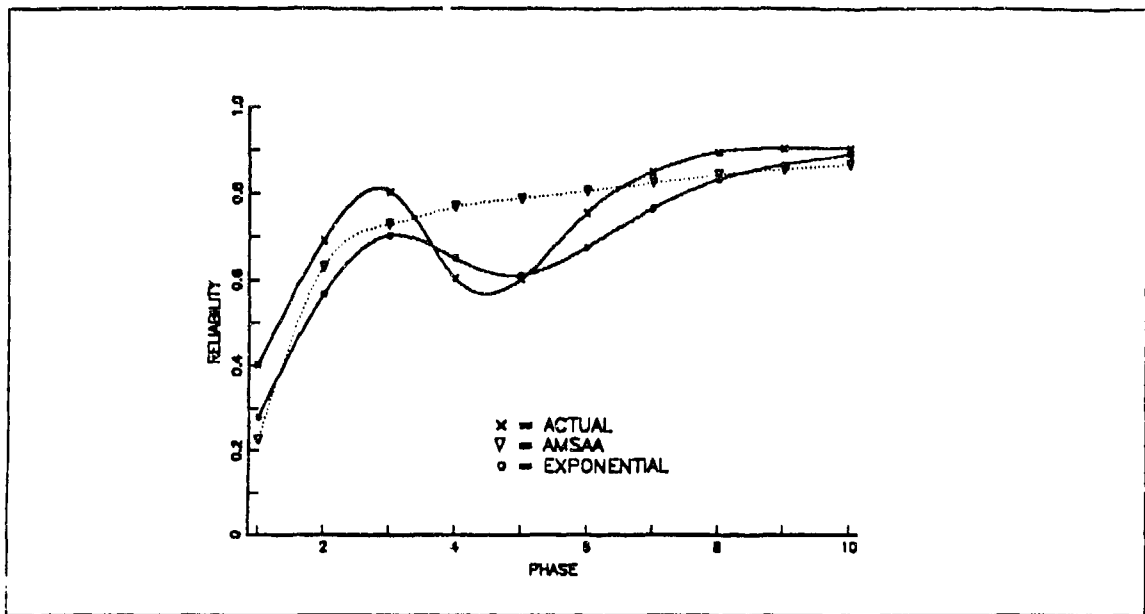


Figure 14. The reliability growth pattern comparison plot case 6

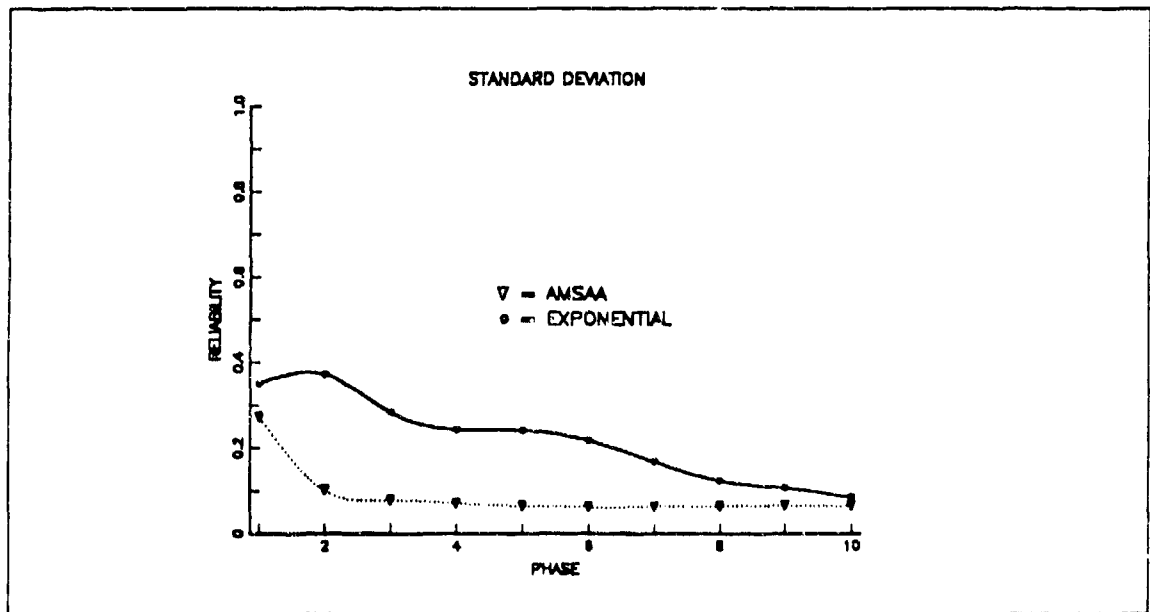


Figure 15. The standard deviation comparison plot case 6

Table 9. STATISTICS FOR CASE 7

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL.	ACTUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	1	.404786	.304970	.319788	.333001	.276939	.262647	.346369
2	1	.598442	.716657	.099573	.725385	.707929	.474285	.378478
3	1	.796763	.811904	.063882	.817504	.806305	.678456	.301077
4	1	.796763	.854719	.050648	.859159	.850280	.747581	.244811
5	1	.802460	.877544	.045115	.881498	.873589	.752545	.242109
6	1	.802460	.891553	.042322	.895262	.887843	.764293	.257599
7	1	.857802	.900743	.041174	.904352	.897134	.816030	.237079
8	1	.902960	.907577	.040321	.911111	.904042	.842211	.241987
9	1	.902960	.912850	.039735	.916332	.909367	.853594	.251411
10	1	.902960	.917156	.038932	.920568	.913743	.855511	.253041



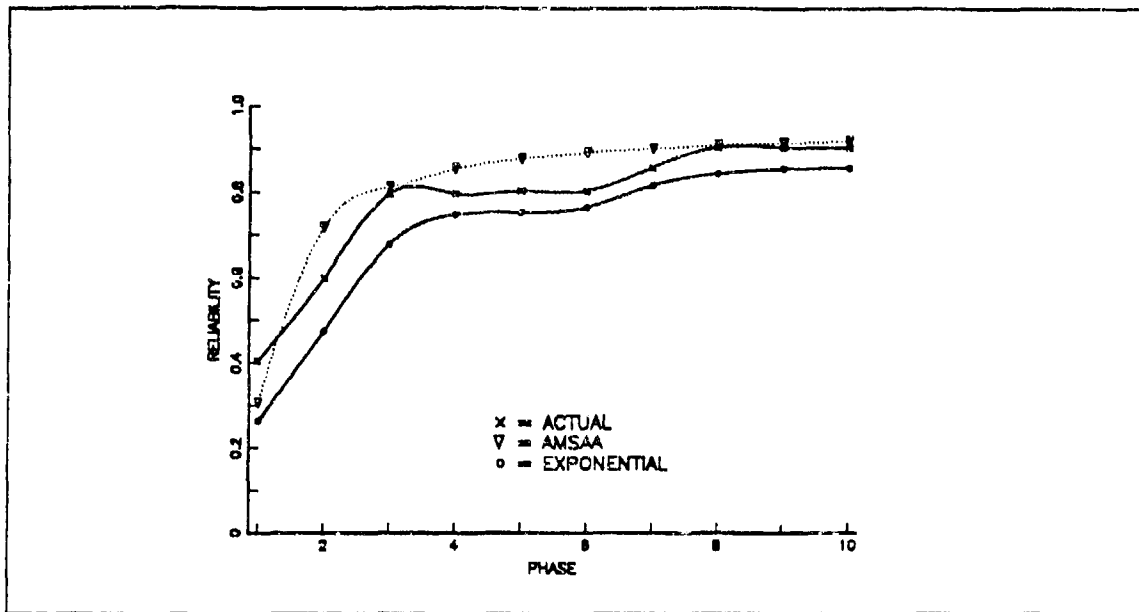


Figure 16. The reliability growth pattern comparison plot case 7

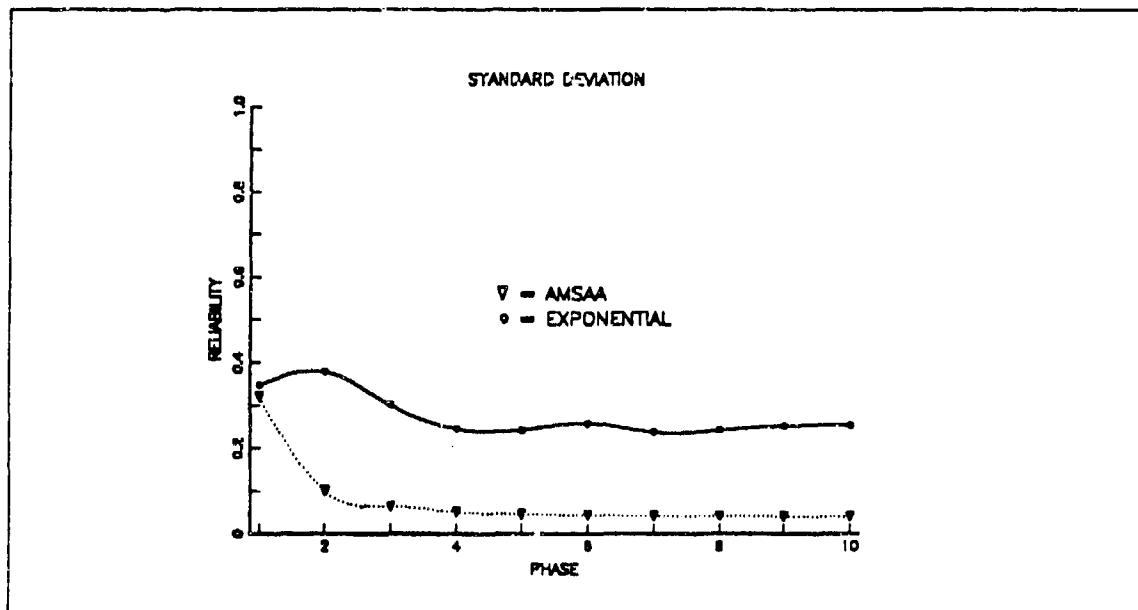


Figure 17. The standard deviation comparison plot case 7

Table 10. STATISTICS FOR CASE 8

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL.	AC- TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	1	.400000	.149057	.192978	.165972	.132142	.271768	.344556
2	1	.430000	.687404	.080545	.694464	.680344	.305861	.350305
3	1	.480000	.769958	.064798	.775638	.764279	.389910	.321783
4	1	.540000	.812713	.056419	.817658	.807767	.449303	.284744
5	1	.610000	.841962	.051100	.846441	.837483	.561109	.261834
6	1	.700000	.864439	.047810	.868630	.860249	.621144	.238394
7	1	.800000	.883576	.045648	.887577	.879575	.712991	.196329
8	1	.900000	.903105	.043961	.906959	.899252	.816562	.138561
9	1	.950000	.922076	.042662	.925816	.918337	.894838	.087652
10	1	.990000	.945008	.042874	.948766	.941250	.959923	.034246

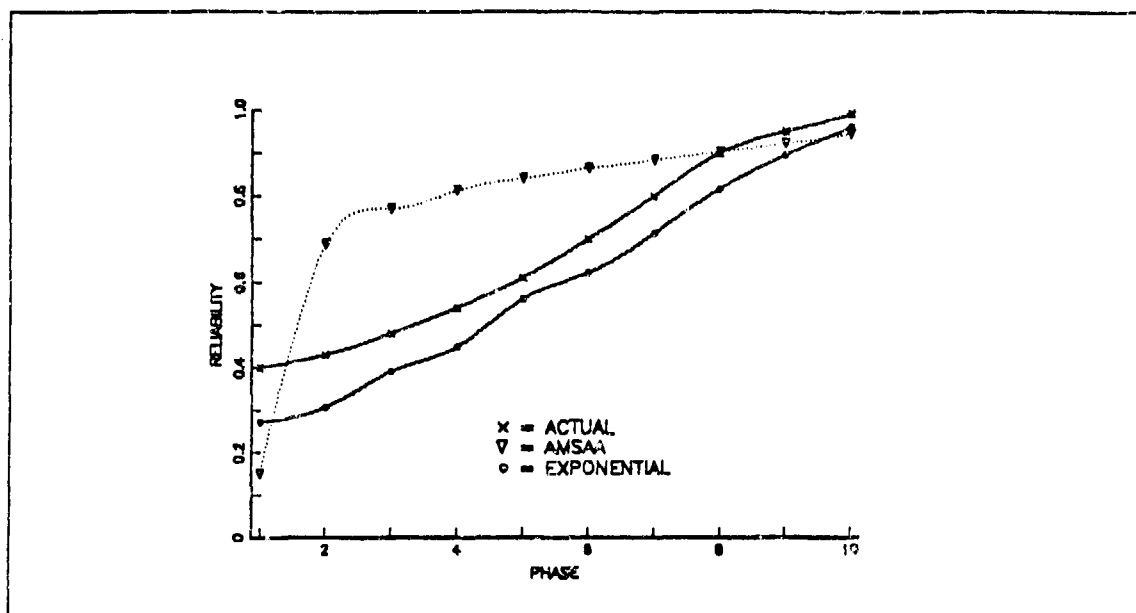


Figure 18. The reliability growth pattern comparison plot case 8

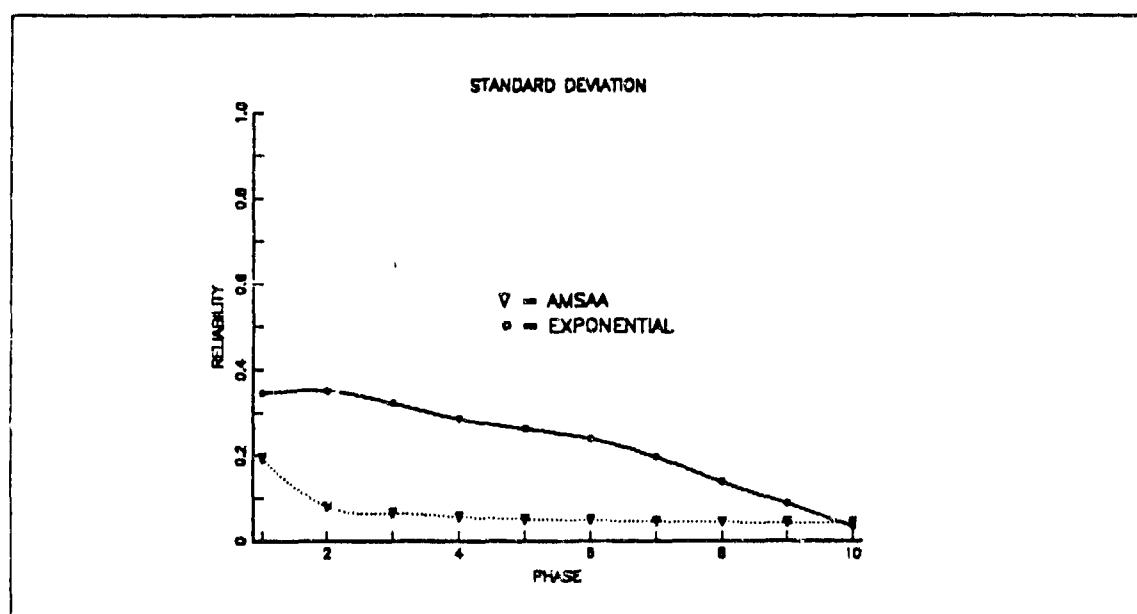


Figure 19. The standard deviation comparison plot case 8

Table 11. STATISTICS FOR CASE 9

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL			EXPONENTIAL MODEL		
	# of FAIL	AC-TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	3	.600463	.538802	.186346	.555136	.522168	.553655	.210395
2	3	.600463	.576379	.086393	.583952	.568807	.563631	.218474
3	3	.600463	.585866	.063621	.591443	.580289	.560046	.196348
4	3	.600463	.590186	.057592	.595234	.585138	.571343	.179061
5	3	.600463	.592796	.058542	.597928	.587665	.583947	.154435
6	3	.600463	.594467	.062527	.599947	.588986	.586601	.147665
7	3	.600463	.595507	.067578	.601431	.589584	.587442	.140020
8	3	.600463	.596269	.072753	.602646	.589891	.586170	.137124
9	3	.600463	.596791	.077717	.603603	.589979	.592676	.123858
10	3	.600463	.596975	.082442	.604201	.589749	.589863	.116577

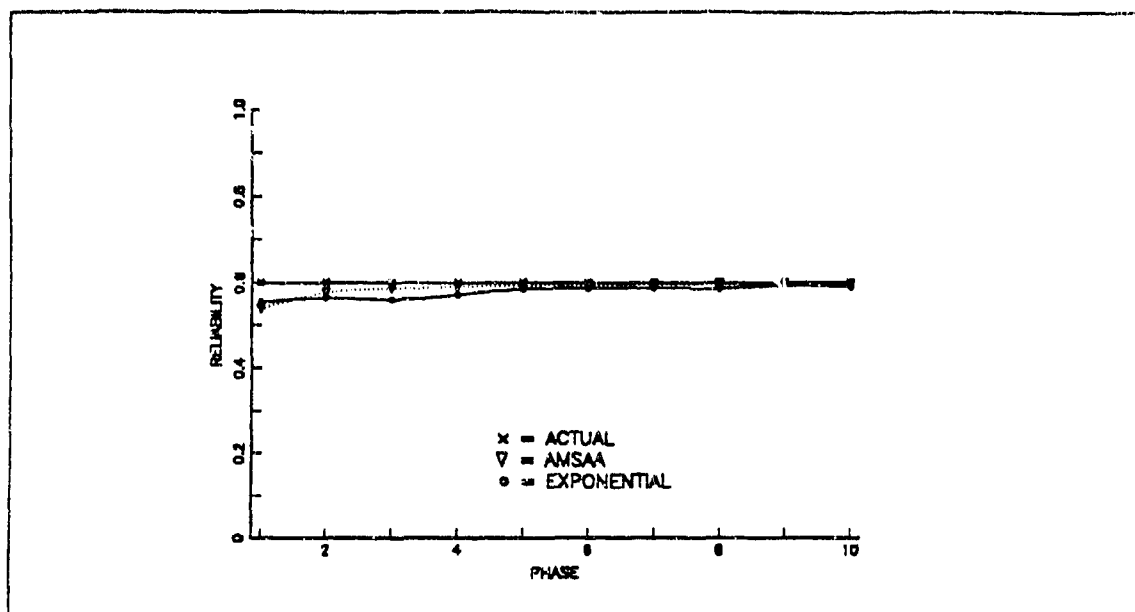


Figure 20. The reliability growth pattern comparison plot case 9

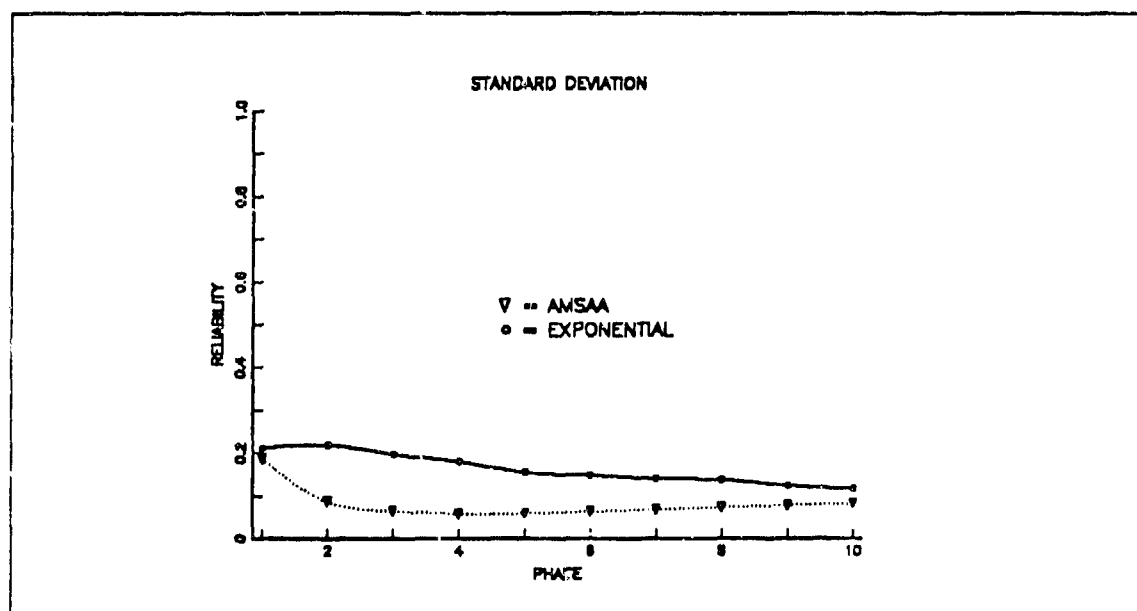


Figure 21. The standard deviation comparison plot case 9

Table 12. STATISTICS FOR CASE 10

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL			EXPONENTIAL MODEL		
	# of FAIL.	AC-TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	3	.403086	.365701	.228152	.385700	.345703	.348426	.225386
2	3	.403086	.398549	.130351	.409974	.387123	.375728	.235090
3	3	.403086	.404570	.096573	.413035	.396105	.378259	.202326
4	3	.403086	.405363	.077487	.412155	.398571	.391223	.180188
5	3	.403086	.404514	.068972	.410559	.398468	.400876	.173112
6	3	.403086	.402537	.070057	.408678	.396397	.403065	.165447
7	3	.403086	.399935	.078909	.406851	.393018	.399075	.157684
8	3	.403086	.397170	.091792	.405216	.389124	.393748	.154780
9	3	.403086	.394409	.106329	.403729	.385089	.400769	.139954
10	3	.403086	.391858	.119968	.402374	.381342	.396248	.130672

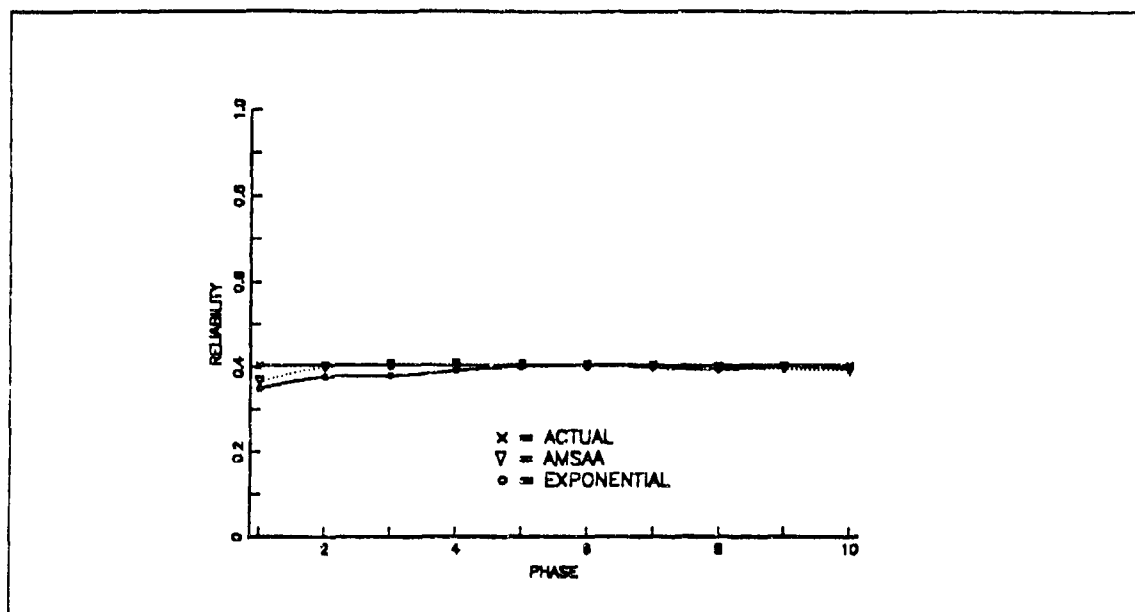


Figure 22. The reliability growth pattern comparison plot case 10

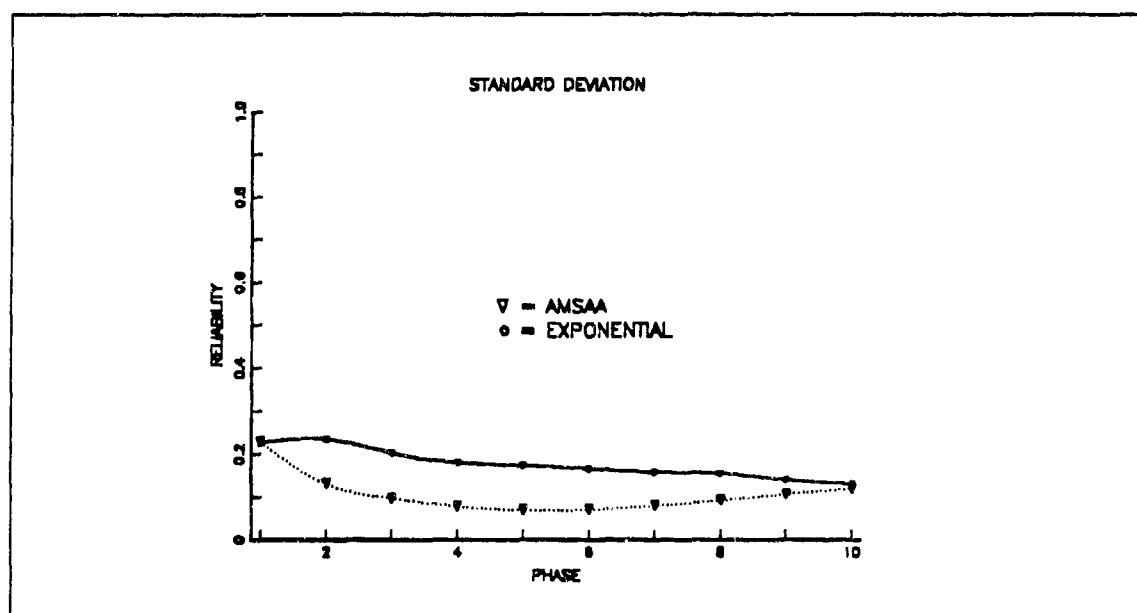


Figure 23. The standard deviation comparison plot case 10

Table 13. STATISTICS FOR CASE 11

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL	AC-TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	3	.899215	.874708	.075400	.881317	.868099	.875154	.097891
2	3	.899215	.890215	.030085	.892852	.887578	.874634	.108637
3	3	.899215	.893745	.022208	.895692	.891798	.881110	.080530
4	3	.899215	.895290	.019809	.897027	.893554	.885150	.071685
5	3	.899215	.896174	.019993	.897927	.894422	.887219	.062968
6	3	.899215	.896677	.021320	.898545	.894808	.889853	.058065
7	3	.899215	.896921	.022891	.898928	.894915	.890704	.054956
8	3	.899215	.897091	.024281	.899219	.894962	.890799	.051037
9	3	.899215	.897191	.026041	.899473	.894908	.893297	.045739
10	3	.899215	.897171	.027662	.899596	.894747	.892541	.043866



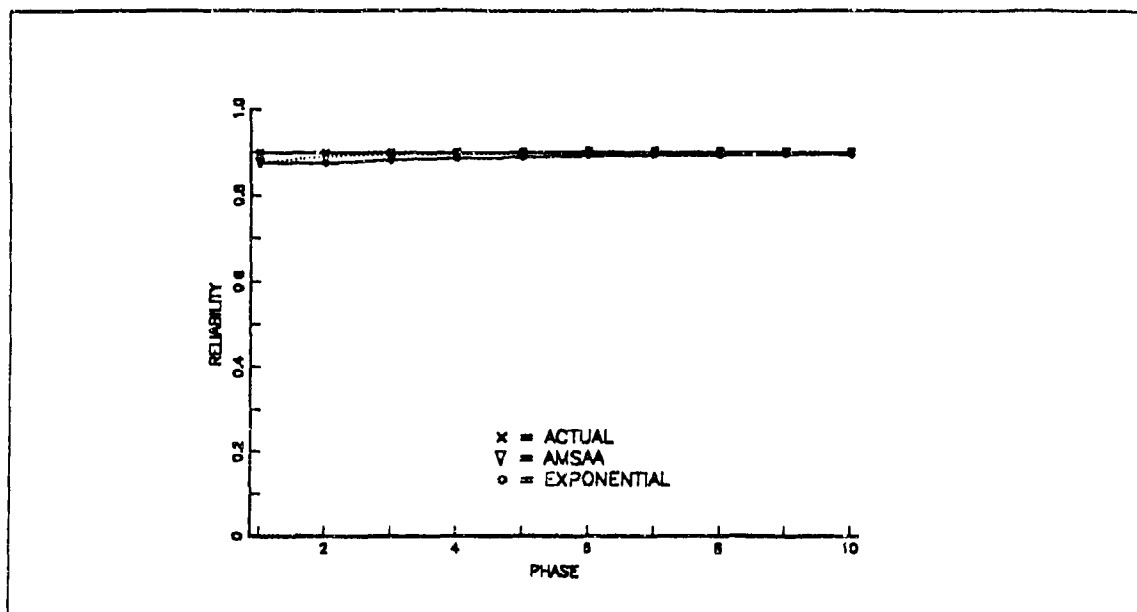


Figure 24. The reliability growth pattern comparison plot case 11

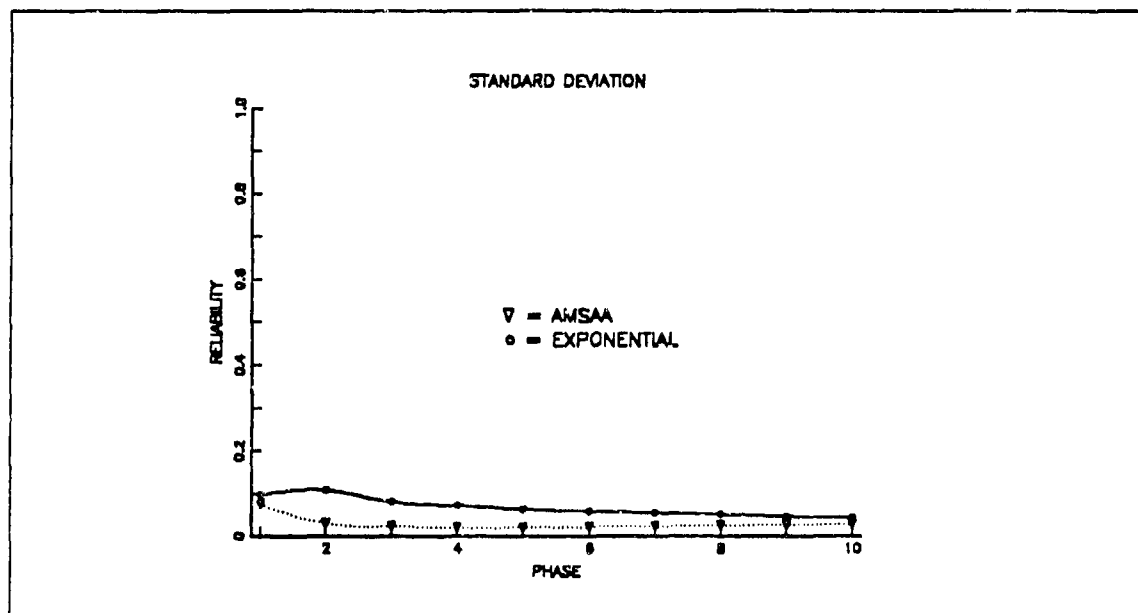


Figure 25. The standard deviation comparison plot case 11

Table 14. STATISTICS FOR CASE 12

PLASID	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL.	AC-TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	3	.403086	.841201	.098829	.849864	.832538	.348426	.225386
2	3	.804723	.849013	.046282	.853069	.844956	.771284	.162229
3	3	.950990	.841209	.050197	.845609	.836809	.939463	.044327
4	3	.975249	.833962	.078297	.840825	.827099	.976039	.017360
5	3	.990040	.830465	.090770	.838422	.822509	.990117	.010184
6	3	.990040	.829695	.091803	.837830	.821561	.993581	.009200
7	3	.990040	.828934	.094842	.837247	.820620	.994806	.008335
8	3	.990040	.828181	.096894	.836674	.819688	.995307	.008680
9	3	.990040	.827444	.098868	.836110	.818778	.995613	.008335
10	3	.990040	.826714	.100896	.835558	.817870	.995641	.008335

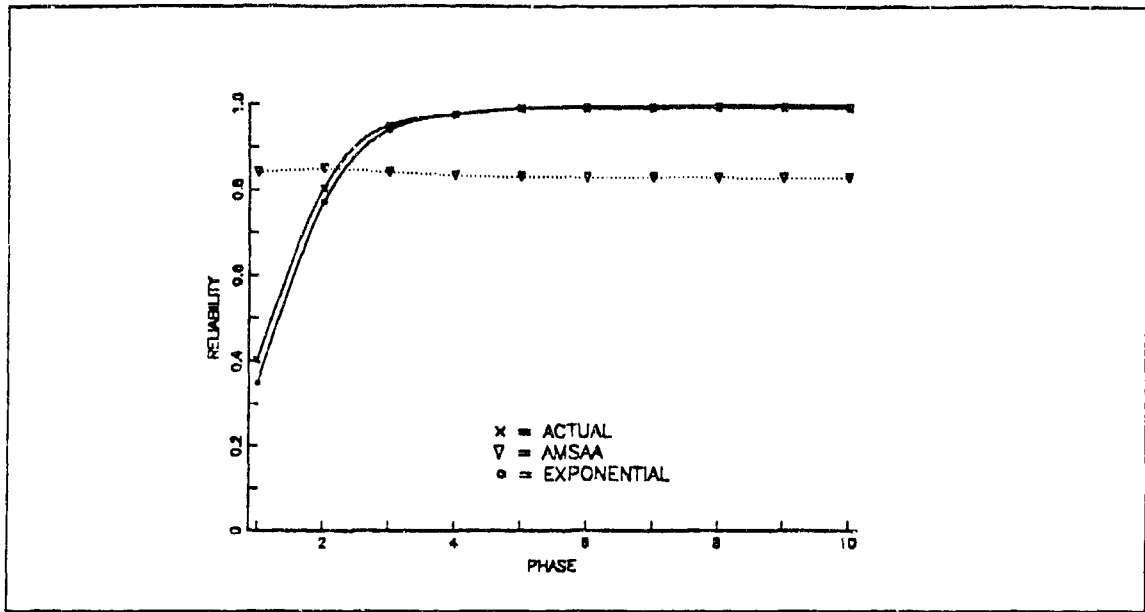


Figure 26. The reliability growth pattern comparison plot case 12

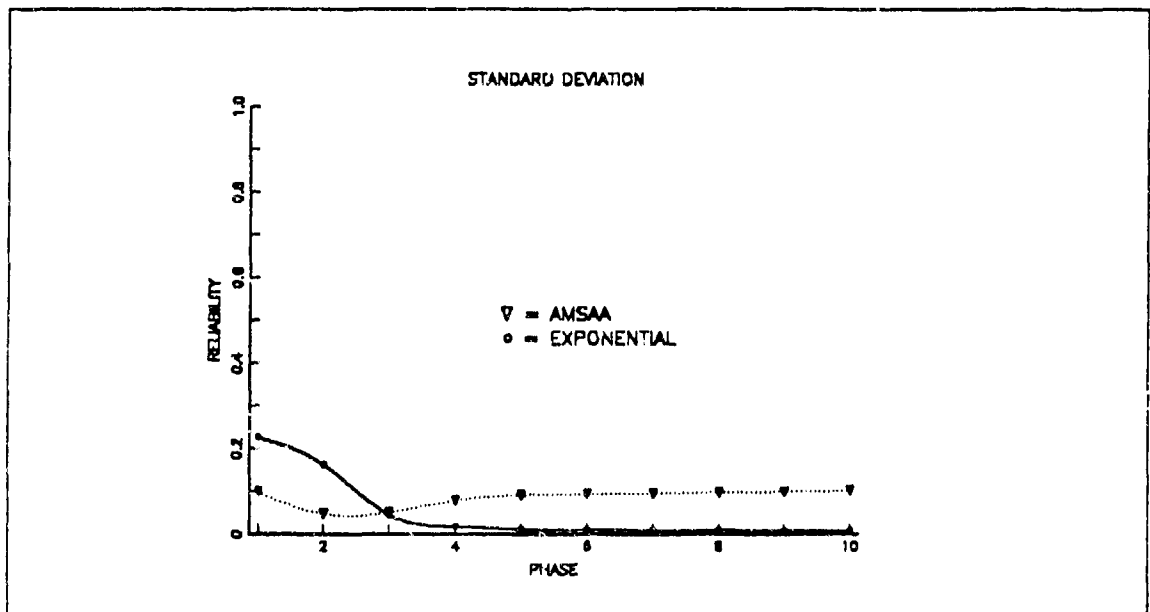


Figure 27. The standard deviation comparison plot case 12

Table 15. STATISTICS FOR CASE 13

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL			EXPONENTIAL MODEL		
	# of FAIL.	AC-TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	3	.403086	.421329	.242162	.442556	.400103	.348426	.225386
2	3	.804723	.751290	.060248	.756570	.746009	.771284	.162229
3	3	.894416	.833149	.034031	.836132	.830166	.887470	.074325
4	3	.899963	.868939	.026237	.871239	.866639	.913950	.052619
5	3	.899963	.887645	.023565	.889711	.885580	.921667	.042965
6	3	.899963	.899931	.022328	.901348	.897433	.924828	.039463
7	3	.899963	.907487	.021942	.909410	.905564	.925187	.037596
8	3	.899963	.913555	.021536	.915443	.911667	.924395	.035405
9	3	.899963	.918287	.021365	.920160	.916414	.925071	.031639
10	3	.899963	.922147	.021066	.923994	.920301	.923207	.031085

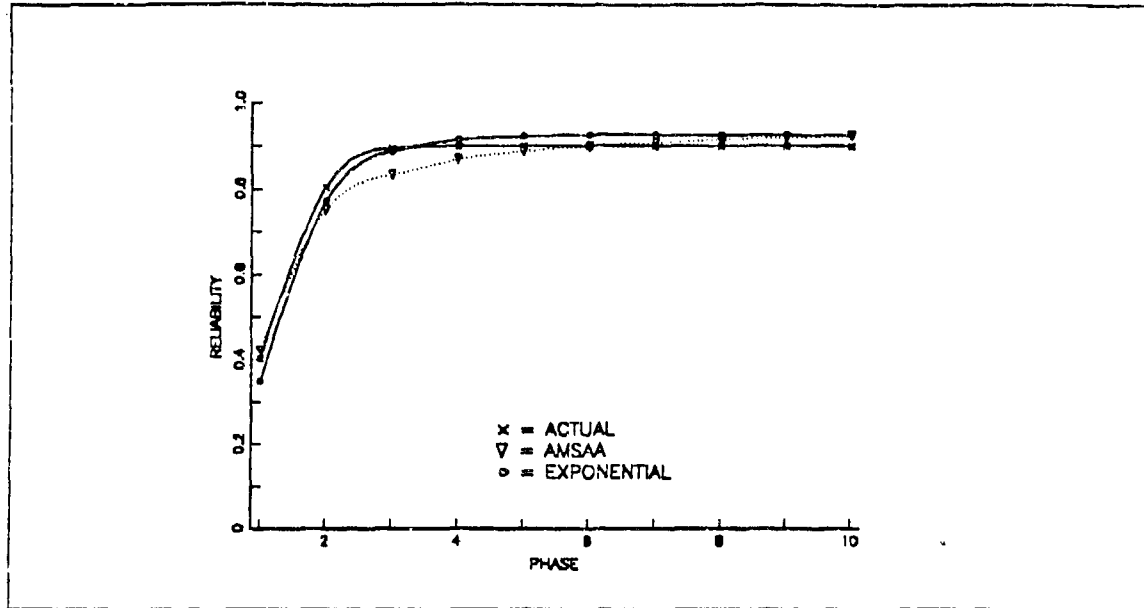


Figure 28. The reliability growth pattern comparison plot case 13

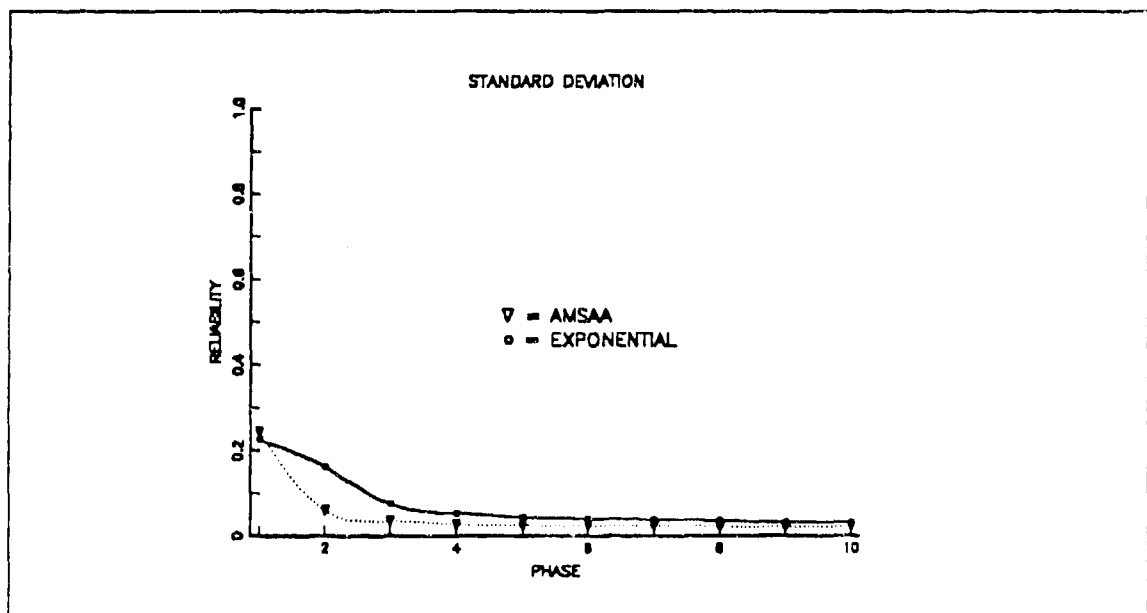


Figure 29. The standard deviation comparison plot case 13

Table 16. STATISTICS FOR CASE 14

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	Job #	AC-TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	3	.403086	.299748	.196088	.316936	.282561	.348426	.225386
2	3	.691333	.659017	.062098	.664460	.653574	.655112	.193326
3	3	.804723	.750623	.043777	.754460	.746785	.788019	.117403
4	3	.603542	.787263	.038458	.790634	.783892	.713682	.126110
5	3	.600463	.803674	.034742	.806720	.800629	.671260	.120385
6	3	.755710	.819347	.032614	.822205	.816488	.720426	.110446
7	3	.849243	.836709	.032380	.839548	.833871	.794972	.084335
8	3	.894416	.853894	.032927	.856780	.851008	.851854	.063108
9	3	.903636	.867828	.033429	.870758	.864897	.886990	.044668
10	3	.903636	.878133	.033186	.881042	.875224	.903086	.038209

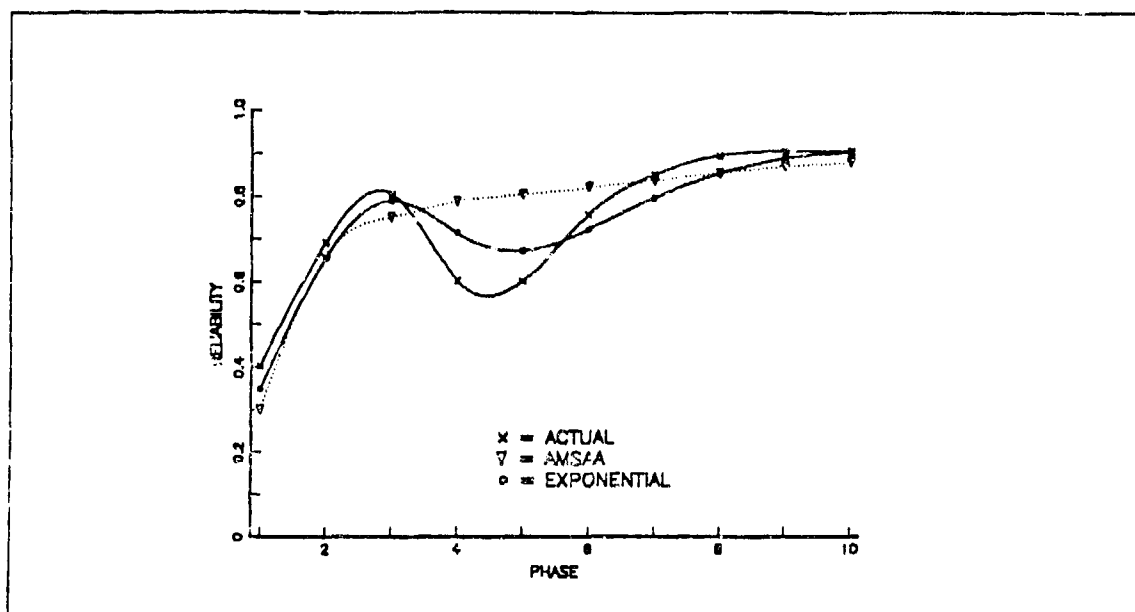


Figure 30. The reliability growth pattern comparison plot case 14

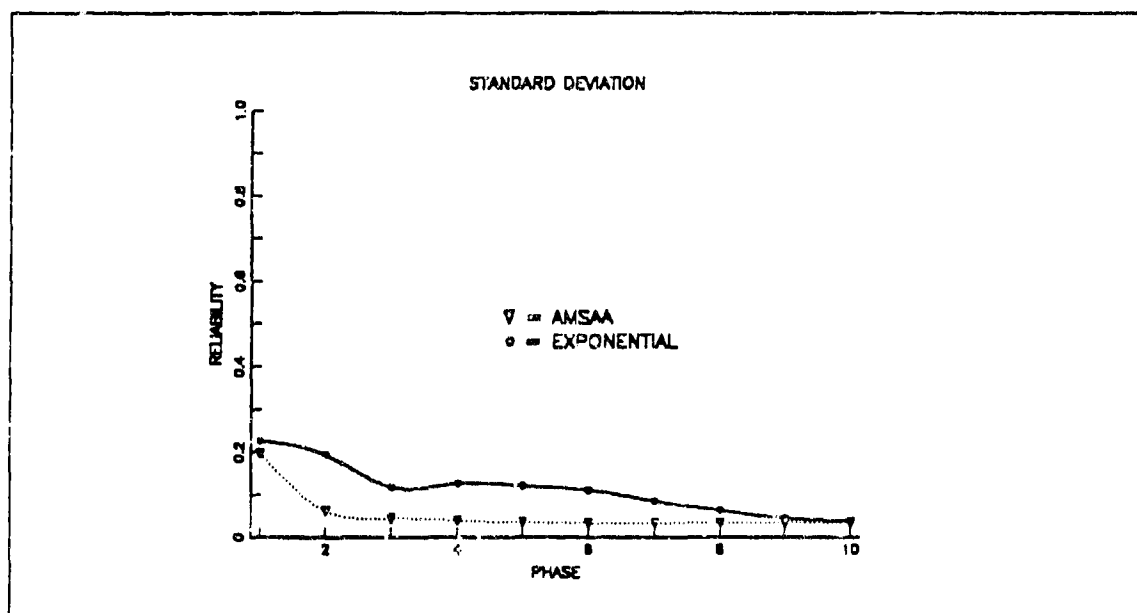


Figure 31. The standard deviation comparison plot case 14

Table 17. STATISTICS FOR CASE 15

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL.	ACTUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	3	.404786	.421329	.242162	.442556	.400103	.351763	.225708
2	3	.598442	.751290	.060248	.756570	.746009	.561042	.219588
3	3	.796763	.833149	.034031	.836132	.830166	.761750	.128729
4	3	.796763	.868939	.026237	.871239	.866639	.807520	.102913
5	3	.802460	.887645	.023565	.889711	.885580	.828525	.080055
6	3	.802460	.899391	.022328	.901348	.897433	.836119	.073760
7	3	.857802	.907487	.021942	.909410	.905564	.860585	.061728
8	3	.902960	.913555	.021536	.915443	.911667	.890172	.048793
9	3	.902960	.918287	.021365	.920160	.916414	.907881	.037583
10	3	.902960	.922147	.021066	.923994	.920301	.915806	.033604



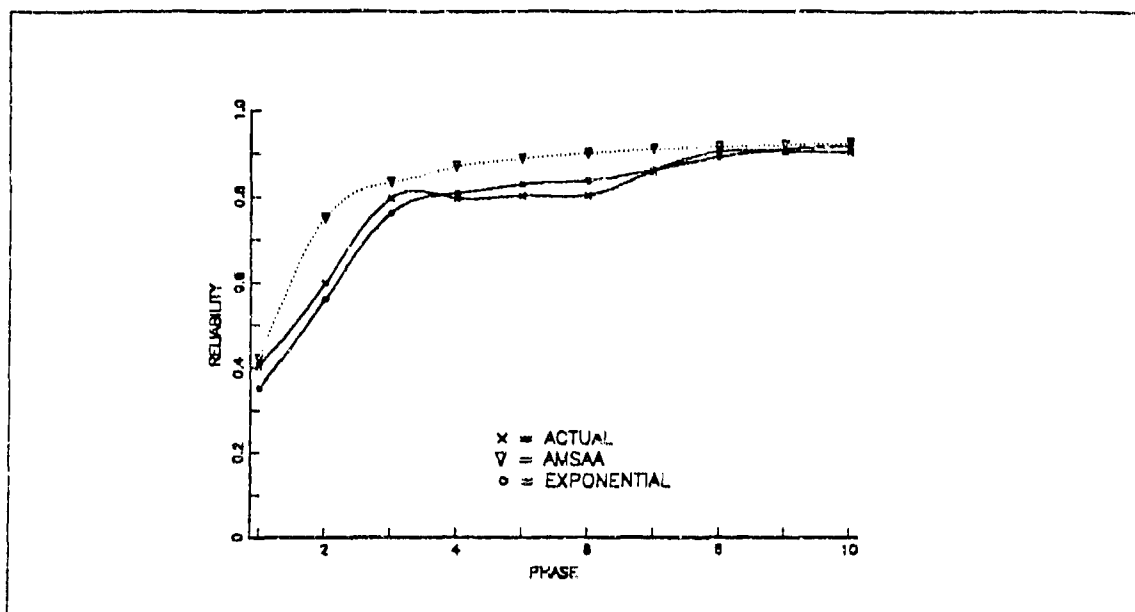


Figure 32. The reliability growth pattern comparison plot case 15

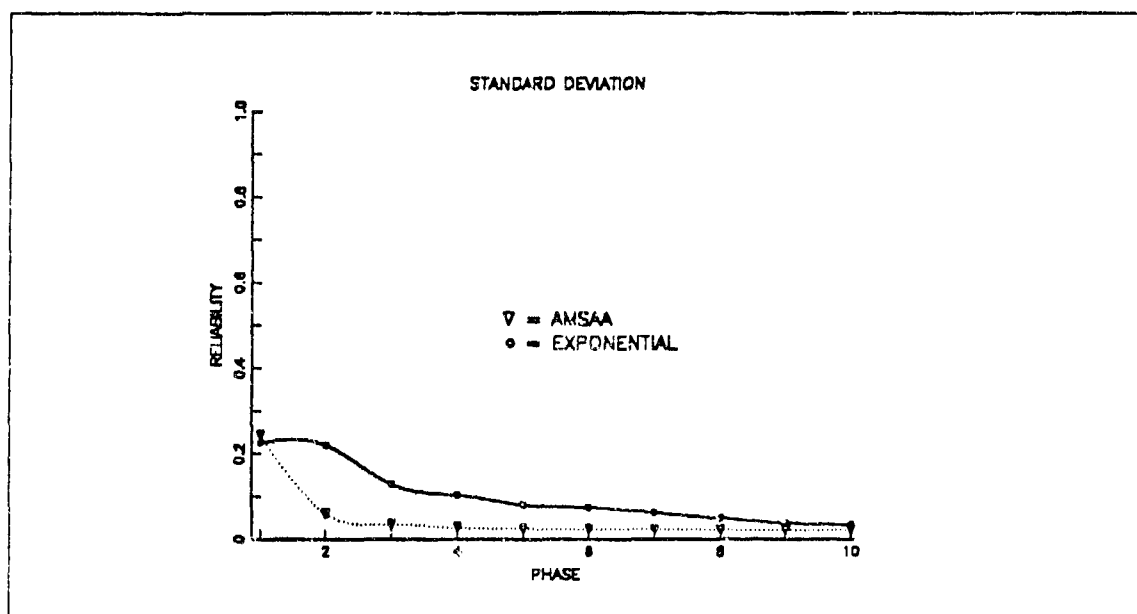


Figure 33. The standard deviation comparison plot case 15

Table 18. STATISTICS FOR CASE 16

PHASE	INPUT DATA		OUTPUT of COMPUTER RUN					
			AMSAA MODEL				EXPONENTIAL MODEL	
	# of FAIL.	AC- TUAL RLBT	MEAN of the EST of RLBT	STD DEV of the EST of RLBT	95% CI of the MEAN of RLBT		MEAN of RGRS EST	STD DEV of RGRS EST
					UPPER	LOWER		
1	3	.400000	.228712	.160711	.242799	.214625	.344437	.225569
2	3	.430000	.594215	.066121	.600011	.588419	.400345	.238088
3	3	.480000	.672784	.053963	.677514	.668054	.444004	.202921
4	3	.540000	.716845	.049957	.721224	.712466	.506867	.180221
5	3	.610000	.748614	.048121	.752832	.744396	.578905	.150044
6	3	.700000	.774650	.047420	.778807	.770494	.659072	.127605
7	3	.860000	.798445	.047698	.802626	.794264	.746996	.099554
8	3	.900000	.824266	.048878	.828550	.819981	.839623	.066243
9	3	.950000	.851483	.050970	.855951	.847016	.909948	.036966
10	3	.990000	.863599	.051552	.868118	.859080	.964163	.016538

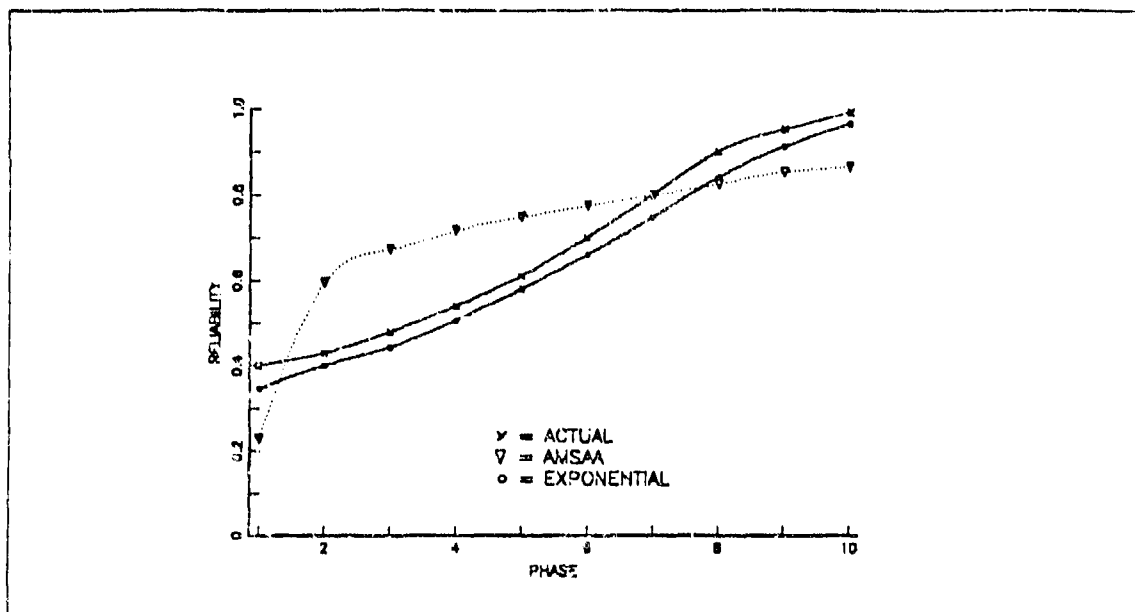


Figure 34. The reliability growth pattern comparison plot case 16

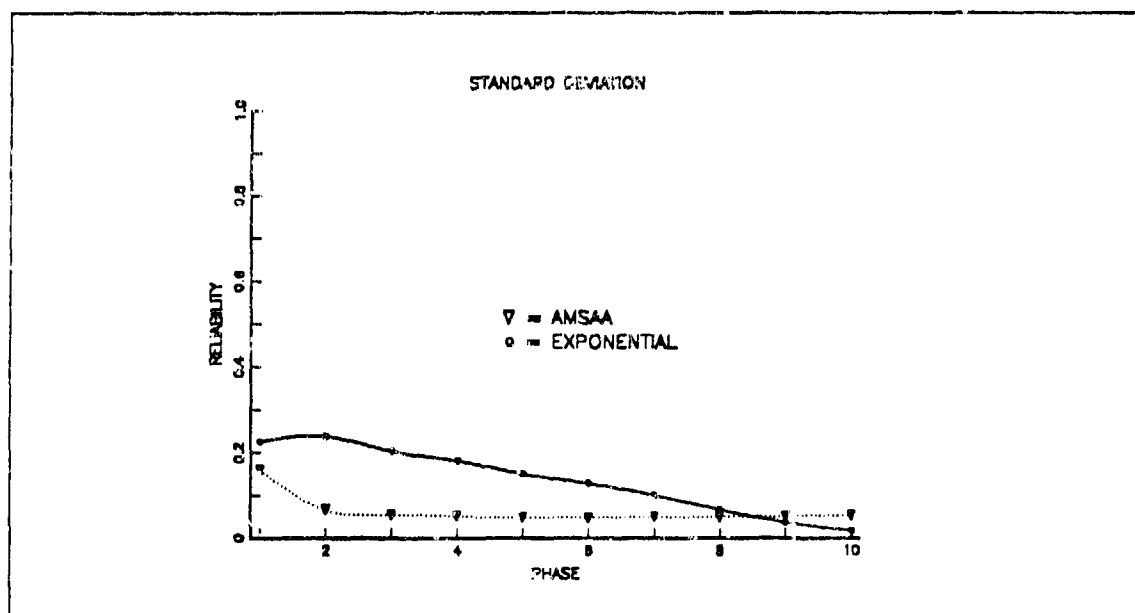


Figure 35. The standard deviation comparison plot case 16

## LIST OF REFERENCES

1. Drake, J. E., Discrete Reliability Growth Model Using Failure Discounting, Naval Postgraduate School, Monterey, California, September 1987.
2. Chandler, J., Estimating Reliability with Discrete Growth Models, Naval Postgraduate School, Monterey, California, March 1988.
3. Corcoran, W. J., and Read, R. R., Comparison of Some Reliability Growth Estimation and Prediction Schemes, United Technology Center Report, Addendum, VTC 2140, 1967.
4. Crow, L. H., AMSAA Discrete Reliability Growth Model, US Army Material System Analysis Activity, Aberdeen Proving Ground, Maryland, 1983.
5. Duane J. T., Learning Curve Approach to Reliability Monitoring, IEEE Transaction On Aerospace, volume 2 number 2, April 1964.
6. Woods, W. M., Reliability Growth Model, paper presented at a meeting of the Avionics Panel held in Ankara, Turkey, 9-13 April 1979.
7. Fishman, G. S., Principle of Discrete Event Simulation, Wiley Series on System Engineering and Analysis, 1978.
8. Lewis, P. A., and Uribe, L., the New Naval Postgraduate School Random Number Package LLRANDOMII, Naval Postgraduate School, Monterey, California, 1981.
9. Miller, A. R., FORTRAN Programs for Scientist and Engineers, Sybex Inc, 1982.

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